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MODEL ARITHMETIC.



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by doing.*



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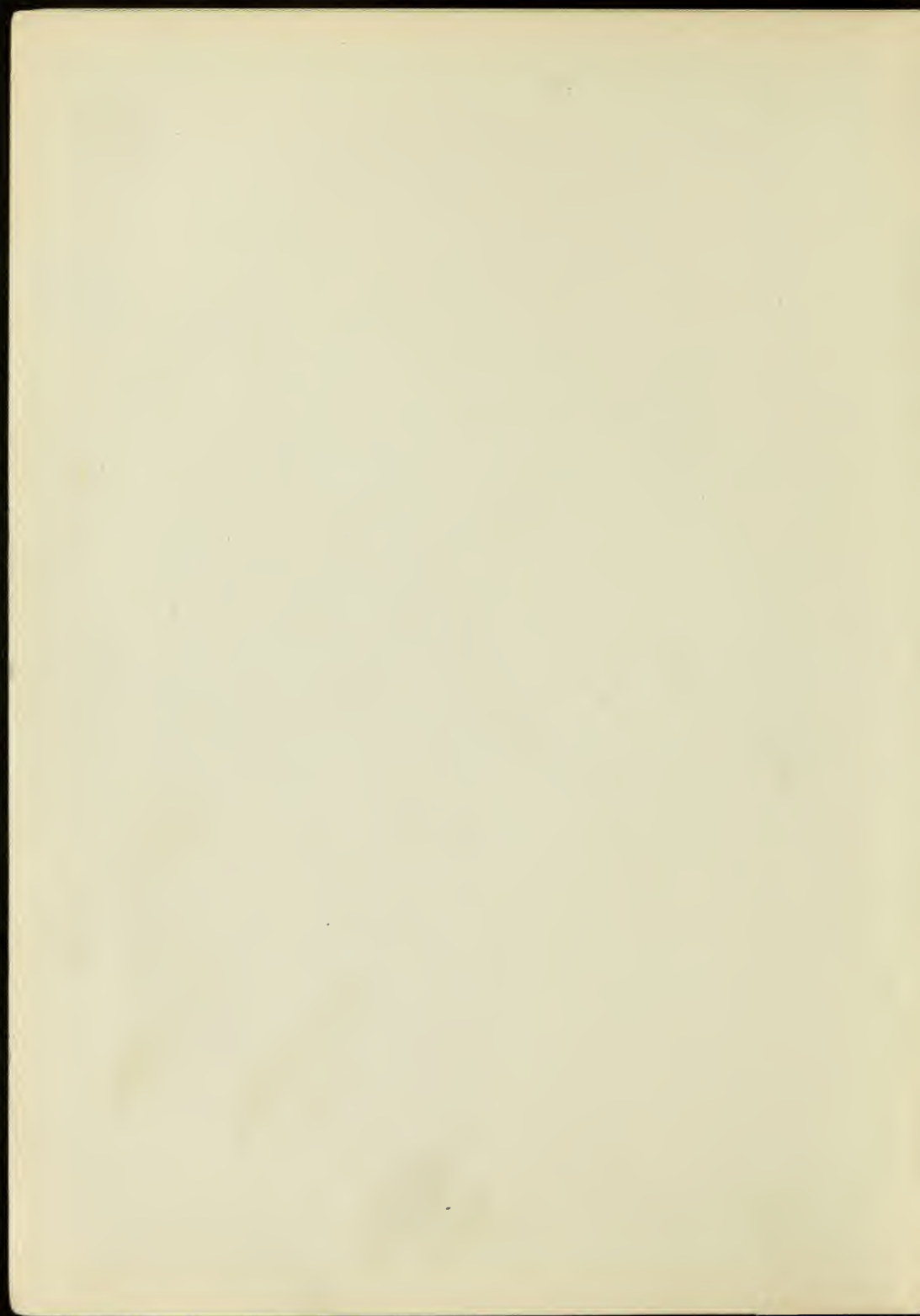
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THE
MODEL ARITHMETIC

FOR
COMMERCIAL COLLEGES

AND
DEPARTMENT SCHOOLS.

A Plain Treatise on the Essentials of Arithmetic, with the Most
Approved Methods of Solution Fully Illustrated.

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PREFACE.

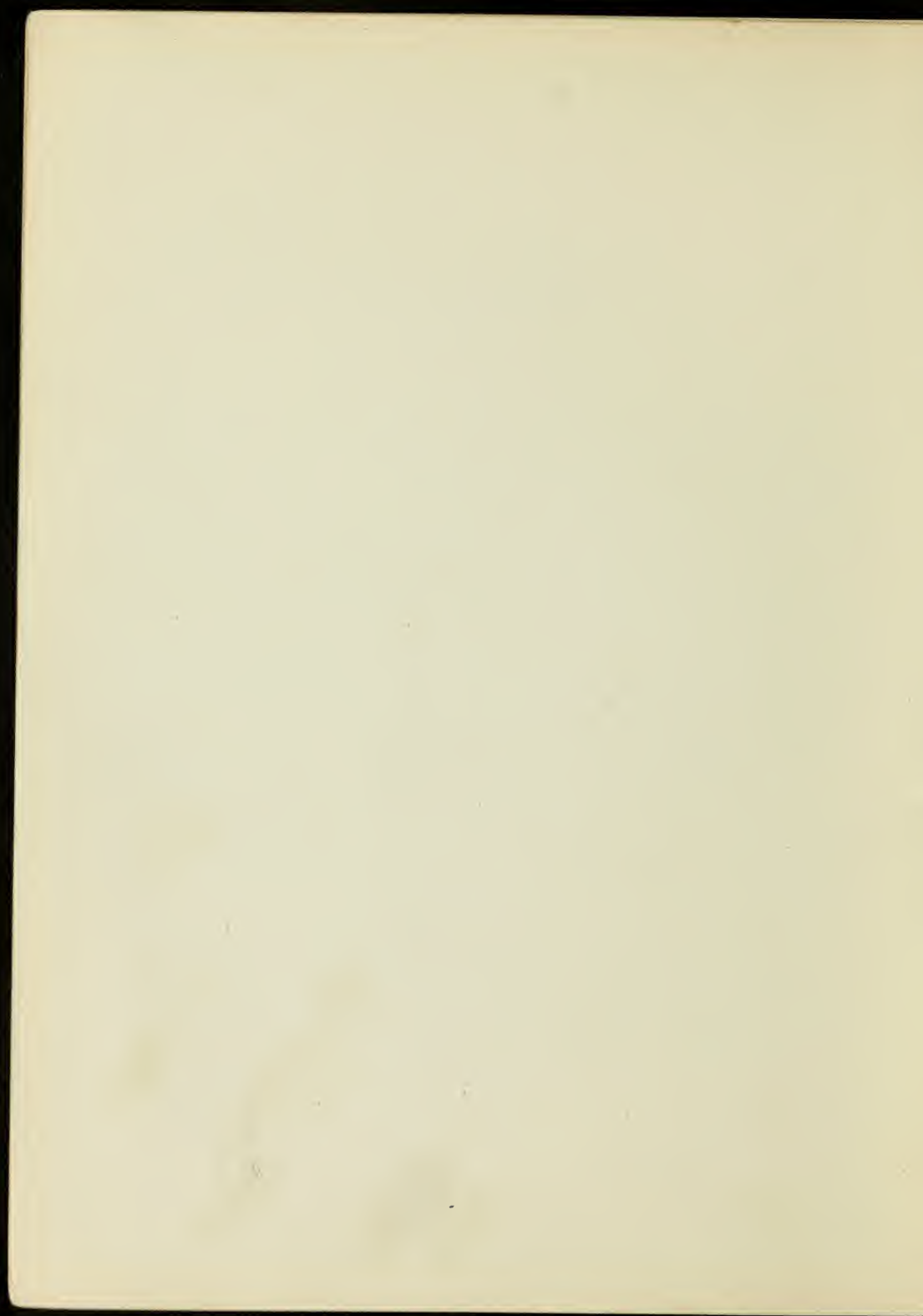
THE call has gone forth for an Arithmetic "without frills," for a book every page of which can be used, and which contains nothing not useful in the every-day practical life of the people. To answer that call this volume has been prepared.

But to prepare such a text-book there has had to be a vigorous use of the pruning knife, among subjects included in the average school textbook on Arithmetic. Very much technical matter, and many topics pertaining alone to abstract mathematics, have been ruthlessly lopped off.

But while limiting the subject-matter of the book to the purely practical, the aim has been, in presenting the selected material, to observe a logical order, and scientific method of treatment.

The book has been written for the student as well as the teacher, and the attempt has been made to elucidate every new subject so plainly that the pupil may be justly held responsible for the preparation of every lesson, without preliminary explanations on the part of the teacher. The book is the exponent of no arithmetical "fads," and where there are several standard methods of solution, all have been given.

While it is not expected that the "Model Arithmetic" will exactly suit every teacher, still it is hoped that the book will not fail of a welcome among those teachers whose classes are expected to reap a bountiful harvest of arithmetical essentials, in the shortest possible allowance of time.



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ARITHMETIC.

GENERAL DEFINITIONS.

1. **Arithmetic** treats of numbers.
2. A **Unit** is a single thing.
3. A **Number** is a Collection of Units or parts of Units, as 5, $\frac{3}{4}$, $7\frac{2}{3}$.
4. An **Integer** is a whole number.
5. An **Even Number** is one that is exactly divisible by 2, as 4, 6, 8, 10.
6. An **Odd Number** is one that is not exactly divisible by 2, as 5, 7, 9.
7. A **Composite Number** is one that can be factored, as, $8 = 4 \times 2$.
8. A **Prime Number** is one that cannot be factored, as 1, 2, 3, 5.
9. A Number is said to be **Abstract** when it is not applied to any class of objects or parts of objects, as 7, $\frac{3}{5}$, $2\frac{1}{4}$, etc.
10. A number is said to be **Concrete** when it is applied to some class of objects or parts of objects, as 7 men, $\frac{3}{5}$ of an acre, $\$2\frac{1}{4}$.

SIGNS.

11. The Sign of Addition is a perpendicular cross +. It is called Plus, and means "add." Thus $6 + 3$ means that 6 is to be added to 3.
12. The Sign of Subtraction is a short horizontal line —. It is called Minus, and means "subtract." Thus, $6 - 2$ means that 2 is to be subtracted from 6.

13. The Sign of Multiplication is an oblique cross \times . It means "multiply by." Thus, 7×6 , means 7 times 6, or that 7 is to be multiplied by 6.

14. The Sign of Division is a short horizontal line placed between two dots \div . It means "divided by." Thus, $15 \div 3$, means that 15 is to be divided by 3.

15. The Sign of Equality is two short horizontal lines $=$. It is read "equals," or "is equal to," and means that the numbers, or quantities, between which it is placed, are equal to each other. Thus $6 + 3 = 9$.

16. Parenthesis (), Brackets [], and Vinculum —, mean that the quantities included within are to be taken together as one quantity.

17. The Decimal Point is a period ($.$), and is used to show the division of a quantity by 10, 100, 1000, 10000, etc. Thus, 2.9 means $29 \div 10$; .67 means $67 \div 100$; .0125 means $125 \div 10000$, etc.

18. An Equation is the expression of equality between two or more quantities, or combinations of quantities. Thus, $(5 + 6) = 11$, $(7 \times 9) = 63$, $(9 + 6) = (10 + 5)$, $\frac{3}{4} = \frac{9}{12}$, are all equations.

NUMERATION AND NOTATION.

19. Numeration is the art of reading numbers.

20. There are three ways of expressing numbers:—

I. By **Words**, as one, two, three.

II By **Figures**, called the **Arabic Notation**, and employing the nine digits.

0 1 2 3 4 5 6 7 8 9

naught one two three four five six seven eight nine

III. By **Letters**, called the **Roman Notation**.

21. In reading numbers in the **Arabic System**, the first figure to the right is called units, the second, tens, the third, hundreds, etc. Each figure to the left increases in value tenfold.

Illustration : $5555 = 5000 + 500 + 50 + 5$.

$37292 = 30000 + 7000 + 200 + 90 + 2$.

22. In order to read numbers greater than hundreds, divide them into periods of three figures each. The first three figures at the right comprise the units' period; the second three figures, the thousands' period; the third, millions; the fourth, billions; the fifth, trillions; the sixth, quadrillions; the seventh, quintillions; the eighth, sextillions; the ninth, septillions; the tenth, octillions, etc.

ILLUSTRATIVE EXAMPLE.

Read, 95364320124.

DIRECTIONS.—Beginning at the right, separate the number into periods of three figures each, as follows: 95,364,320,124. As each period is pointed off, think of its name: as, first, units; second, thousands; third, millions; fourth, billions; now, read each period as if it stood alone, giving its name, and you have, 95 billion, 364 million, 320 thousand, 124.

NOTE 1.—Observe that the value of a number is not affected by ciphers preceding it: thus, 006 = 6; 010 = 10; 069 = 69. This must be borne in mind while reading the periods. Notice also that each period *except the lefthand one* MUST consist of **THREE FIGURES**.

NOTE 2.—The figures of a number are sometimes called *Digits*. By the *Order* of a figure, or digit, is meant the number of its position from the right. Thus, 5267 = 5 units of the fourth order, 2 units of the third order, 6 units of the second order, and 7 units of the first order. Ciphers are used to show the absence of orders. Thus, in 506 the cipher shows that there are no units of the second order in this number.

After carefully considering the above explanation, read the following numbers :—

1. 264.	8. 8202645.	15. 50001.
2. 954.	9. 26342109.	16. 151316000.
3. 3664.	10. 9002484.	17. 4005006.
4. 56321.	11. 10400064.	18. 300204017.
5. 70324.	12. 300036.	19. 10001.
6. 216395.	13. 402301.	20. 10001000.
7. 1002006301.	14. 9002036000.	21. 50000006.

23. Notation is the art of writing numbers.

ILLUSTRATIVE EXAMPLE.

Express by figures, four hundred six million, forty-one thousand, **five** hundred sixty-two.

DIRECTION.—Write the digits necessary to express the lefthand period; then observe that, as “forty-one” consists of but two figures, and as this period must consist of three figures, a cipher must precede the “forty-one,” thus, 041. Now, write the next lower period, 562, and we have the entire number, 406041562.

In the same manner write in figures the following numbers:—

1. Three thousand, one hundred forty-eight.
2. Two thousand, one hundred forty-seven.
3. Sixty-five thousand, three hundred eighty-one.
4. Sixty-four thousand, one hundred eighty-one.
5. Ninety-four thousand, twenty-four.
6. Two hundred seventeen thousand, thirty-four.
7. One hundred thousand, two hundred two.
8. Three million, one hundred thousand, one.
9. Ten thousand, one.
10. One hundred one thousand, eleven.
11. Six trillion, four hundred million, three hundred one.
12. Ninty-six trillion, fourteen thousand, two.
13. Fourteen billion, three hundred six thousand.
14. Thirty-two quintillion, six trillion, forty-four thousand, **one** hundred one.

THE ROMAN SYSTEM.

24. The **Letters** employed in the **Roman Notation** are, I for one, V for five, X for ten, L for fifty, C for one hundred, D for five hundred, and M for one thousand.

25. Repeating a letter repeats its value. As, X equals ten, XX will equal twenty.

26. Annexing a letter to one of greater value indicates the sum of the two values; but, if a letter is prefixed to one of greater value, it indicates the difference of their values. Thus, VI equals 6, IV equals 4, XC equals 90, CX equals 110.

27. A dash —, placed over a letter, increases its value one thousand-fold. Thus, V equals five, \overline{V} equals five thousand, CM equals nine hundred, \overline{CM} equals nine hundred thousand.

28. Several dashes may be used, each multiplying the number by 1000; thus, $\overline{\overline{X}}$ equals ten thousand, $\overline{\overline{\overline{X}}}$ equals ten million, and $\overline{\overline{\overline{\overline{X}}}}$ equals ten billion, etc.

TABLE OF ROMAN NUMBERS WITH ARABIC EQUIVALENTS.

I,	1	XII,	12	L,	50	DCC,	700
II,	2	XIII,	13	LX,	60	DCCC,	800
III,	3	XIV,	14	LXX,	70	CM,	900
IV,	4	XV,	15	LXXX,	80	M,	1000
V,	5	XVI,	16	XC,	90	MM,	2000
VI,	6	XVII,	17	C,	100	\overline{X} ,	10000
VII,	7	XVIII,	18	CC,	200	\overline{L} ,	50000
VIII,	8	XIX,	19	CCC,	300	\overline{C} ,	100000
IX,	9	XX,	20	CD,	400	\overline{D} ,	500000
X,	10	XXX,	30	D,	500	$\overline{\overline{M}}$,	1000000
XI,	11	XL,	40	DC,	600		

ILLUSTRATIVE EXAMPLES.

1. Write 2645 by the Roman Method.

DIRECTIONS.—First, write its highest order (thousands), thus, MM, then the next highest order (hundreds), thus, DC, then the next order (tens), thus, XL, and finally the lower order (units), thus, V; then combine these and the result is MMDCXLV.

2. Write 5,256,519.

DIRECTIONS.—Write 5 million thus, $\overline{\overline{\overline{V}}}$, 256 thousand thus, \overline{CCLVI} , 519 thus, DXIX, or = $\overline{\overline{\overline{V}}CCLVI}DXIX$.

Examine carefully the above explanations then read the following expressions:—

XVI.	XXIV.	XLI.	LXXIX.	XCIH.
CCIV.	DCX.	DCCLXIV.	MXX.	CMLXIV.
MDC.	MDCCCXCIV.	MC.	CLX.	\overline{LXV} .
\overline{CD} .	\overline{XI} .	\overline{XIV} .	\overline{DCX} .	\overline{CM} .
\overline{V} .	$\overline{\overline{XLIV}}$.	$\overline{\overline{LXIXXII}}$.	$\overline{\overline{\overline{VXVLIH}}}$.	$\overline{\overline{\overline{\overline{XIXCCDC}}}}$.

Write by the Roman Method the following numbers :—

8450.	5160.	92045.	11087.	253620.
5425.	6005.	18650.	123456.	910732.
8595.	2100.	57060.	47021.	1258746.
7963.	9000.	18540.	82176.	29000410.
3021.	75634.	67039.	570914.	14010658246.

ADDITION.

29. Addition is the process of uniting two or more numbers into one number.

30. The Sign of Addition is $+$, and means that the numbers between which it is placed are to be added.

31. The Addends are the numbers to be added.

32. The Sum, or Amount, is the result obtained by addition.

The numbers must be placed in their regular order, that is, units under units, tens under tens, etc.

33. In adding use the Group Method. As for example :—

9 6 7 4 6 8 2 7 2 4 3 1

Say, 9, 22, 32, 42, 51, 59, then, add in the same way from right to left.

34. Use the Group Method whether the numbers are arranged horizontally or perpendicularly. Larger groups may be made, as proficiency is acquired. Never repeat the numbers you are adding; simply the result.

35. It is very important that the student should be rapid and accurate in addition. One small error in making out a bill, may make a difference of several hundred dollars. First be accurate, and then, by much practice, rapidity will come.

36. It is also important to be able to *announce instantly* the unit figure in the result of any addition. Beginners will often at once give the

result of any combination of one figure with another, as $6 + 8$, $9 + 6$, $8 + 3$, etc., but will hesitate, or even count on their fingers, in getting the result of $26 + 8$, $49 + 6$, $68 + 3$, etc. To correct this difficulty, the student should know that $6 + 8$ always gives 4, as the unit figure, whether either the 6 or the 8 is, or is not, preceded by another figure or figures. This knowledge should be mechanical, and should be secured by thorough and continued drill. Following are a few suggestive drills which may be varied indefinitely by the teacher. Advanced pupils are often wretchedly deficient in addition, because the subject has been neglected in their early training. We recommend daily drills in these and similar exercises, until the pupils can state instantly any possible combination of unit figures.

EXERCISE I.

Announce the sum of:—

$27 + 9$	$27 + 7$	$79 + 6$	$64 + 6$	$743 + 8$	$427 + 4$
$64 + 7$	$54 + 8$	$44 + 9$	$49 + 7$	$128 + 7$	$586 + 8$
$47 + 8$	$59 + 6$	$87 + 5$	$98 + 8$	$658 + 3$	$174 + 6$
$63 + 9$	$29 + 5$	$63 + 7$	$129 + 6$	$126 + 6$	$295 + 8$
$48 + 7$	$42 + 9$	$59 + 2$	$325 + 9$	$247 + 6$	$537 + 9$
$65 + 6$	$53 + 8$	$89 + 9$	$146 + 8$	$859 + 3$	$128 + 7$

EXERCISE II.

Begin with 6 and count by 9 to 100. Thus, 6, 15, 24, 33, 42, etc.

Begin with 4, count by 6 to 100.	Begin with 3, count by 7 to 100.
Begin with 7, count by 4 to 100.	Begin with 2, count by 9 to 100.
Begin with 5, count by 8 to 100.	Begin with 8, count by 7 to 100.

37. The Complement of a Number is the number that must be added to it to produce a unit of the next higher order. Thus, the complement of 6 is 4, of 7 is 3, of 29 is 71, of 63 is 37, of 246 is 754, etc.

NOTE.—It will be noted that in complementary numbers, the two unit figures added equal 10, while the other figures of the same order added equal 9. Thus, 346 and 654 are complements.

Adding we have	3 4 6	Also	7 5 2 8
	<u>6 5 4</u>		<u>2 4 7 2</u>
	9 9 10 or 1000,		9 9 9 10 or 10000. etc.

EXERCISE III.

State the complement of the following :—

65	59	84	48	82	126	743	327	591
73	38	66	41	71	348	239	456	842
46	27	51	52	56	527	651	382	325

RECORDING PARTIAL ADDITIONS.

38. Clerks, bookkeepers, and other accountants are apt to be interrupted in their work, when a column of figures is almost added, and may have to do all their work over. The following device obviates this by recording the work already done.

\$ 24.52	25	EXPLANATION.—The first column to be added amounts to 25, which amount is set down; the next column is added and placed under it, one figure to the left, and so on. This leaves the work in a position to be verified at any time.
26.78	28	
29.21	42	
46.48	15	
29.42		
38.64		
<hr/> \$195.05	19505	

PROOF OF ADDITION.

39. Here is a simple and practical method of proving addition :—

1864 = 19 = 10 = 1	EXPLANATION.—Add the figures in each line horizontally. The first line adds to 19, and the sum of these two figures is 10, and the sum of these is 1. The second line adds to 18, the sum of these two figures is 9. The third adds to 18, and 8 + 1 is 9. The fourth adds to 8. The fifth adds to 27, and 7 + 2 is 9. In each case keep on adding until you have reduced the line to one figure, or unitate. Now find the sum of all these, which is 36, and 3 + 6 is 9, the final unitate. Then add the figures in the answer, 1 + 0 + 3 + 9 + 5 = 18; 8 + 1 = 9, the final unitate. If the addition is correct, the final numbers or unitates will be the same. After a little practice the student will find that he can set down the final unitates, omitting all intervening figures.
1953 = 18 = 9	
1764 = 18 = 9	
1025 = 8	
3789 = 27 = 9	
<hr/> 10395 = 18 = 9. 36 = 9.	

Another convenient method of proof consists in adding the figures of a column a second time in different order, that is, begin at the

bottom of the column in the first addition, and at the top in the second. The results, of course, should be the same.

EXAMPLES FOR PRACTICE, USING GROUP METHOD.

(PROVE YOUR WORK.)

Add :—

1. 21689	2. 65473	3. 74688	4. 56432156
42384	45548	15443	75439965
56849	37189	72792	28365283
13489	83326	83327	96543216
65964	54275	64654	76896543
78654	76435	58433	76987654
38265	89667	78522	76345923
54382	22143	91465	91683468
28654	77798	25549	23456837

Add the following example vertically, and then horizontally. The sums of the totals should coincide in the final total.

5. Grain purchased by Grain Buyer for one week :

	Monday.	Tuesday.	Wednesday.	Thursday.	Friday.	Saturday.	Total.
Wheat,	23,124	15,165	19,176	17,849	65,384	74,396
Corn,	75,384	36,785	16,384	39,756	38,695	37,965
Oats,	13,750	29,387	16,384	39,756	18,975	13,864
Rye,	17,386	37,485	38,396	64,587	29,654	9,698
Barley,	36,184	7,984	9,387	9,569	8,675	9,324
Total,

6. Find the total population of the United States for 1890.

Ala.,	1,513,017	D. C.,	440,405	Kan.,	1,423,485
Alaska,	31,795	Fla.,	391,422	Ky.,	1,855,436
Ariz.,	59,620	Ga.,	1,837,353	La.,	1,116,828
Ark.,	1,128,179	Idaho,	84,385	Maine,	661,086
Cal.,	1,208,130	Ill.,	3,826,351	Md.,	1,040,431
Colo.,	412,198	Ind.	2,192,404	Mass.,	2,238,943
Conn.,	746,258	Ind. T.,	87,699	Mich.,	2,089,792
Del.,	168,493	Iowa,	1,906,729	Minn.,	1,300,017

Miss.,	1,284,887	N. C.,	1,617,340	Tenn.,	1,763,723
Mo.,	2,677,080	N. Dak.,	182,719	Tex.,	2,232,220
Mont.,	131,769	Ohio,	3,693,216	Utah,	206,498
Neb.,	1,056,793	Okla.,	18,852	Vt.,	332,422
Nev.,	45,761	Ore.,	312,490	Va.,	1,648,911
N. H.,	376,530	Pa.,	5,258,014	Wash.,	349,516
N. J.,	1,441,017	R. I.,	345,343	W. Va.,	762,794
N. M.,	144,862	S. C.,	1,151,149	Wis.,	1,683,697
N. Y.,	5,981,934	S. Dak.,	327,848	Wyo.,	60,705

SUBTRACTION.

40. Subtraction is the process of finding the difference between two numbers

41. The two numbers in subtraction are called respectively the **Minuend** and **Subtrahend**.

42. The **Minuend** is the number from which the other is to be taken.

43. The **Subtrahend** is the number to be subtracted.

44. The **Difference**, or **Remainder**, is the result.

45. The **Sign of Subtraction** is —, and is called **Minus**.

PROOF OF SUBTRACTION.

46. Add the **Remainder** to the **Subtrahend**; if the work be correct, the sum will equal the **Minuend**; or, subtract the **Remainder** from the **Minuend**; if the work be correct, the difference will equal the **Subtrahend**.

EXAMPLES FOR DRILL.

Find the difference between —

- | | |
|---------------------------|------------------------------------|
| 1. \$3,640 and \$2,160. | 7. \$36,149 and \$12,964. |
| 2. \$2,360 and \$1,975. | 8. \$91,786 and \$43,597. |
| 3. \$12,386 and \$9,389. | 9. \$368,110 and \$154,965. |
| 4. \$56,789 and \$9,678. | 10. \$1,618,125 and \$935,687. |
| 5. \$36,141 and \$28,691. | 11. \$30,050,067 and \$15,609,849. |
| 6. \$31,684 and \$21,496. | 12. \$56,004,201 and \$49,658,256. |

GENERAL PROBLEMS.

1. A man traveled 340 miles on Monday, 541 miles on Tuesday, 384 miles on Wednesday, 921 miles on Thursday, 724 miles on Friday, and 396 miles on Saturday. Find total number of miles traveled.

2. A farmer has 1986 bushels of wheat, 3240 bushels of oats, 5648 bushels of corn, and 1324 bushels of rye. Required the total number of bushels.

3. A man bought a house for \$7,630, paid \$524 for painting, \$340 for repairs, and then sold it at a profit of \$1,256. What did he receive for the house?

4. A grocer's sales were as follows: Monday, \$652; Tuesday, \$341; Wednesday, \$748; Thursday, \$391; Friday, \$548; Saturday, \$674. What were his total sales for the week?

5. A man bought a lot for \$6,410, paid \$510 for grading, \$65 to have it enclosed, and then sold it at a gain of \$450. How much did he receive for the lot?

6. A man bought five lots and paid \$7,560 for them; for the first he paid \$1,200; for the second, \$1,750; for the third, \$940; and for the fourth \$1,220. What did the fifth lot cost him?

7. A man deposited \$5,600 in the bank; on Monday he drew out \$240; on Tuesday, \$1,540. How much has he still on deposit?

8. A real-estate agent bought a house for \$20,000; paid out \$310 for repairs and then sold it for \$22,125. What was his gain?

9. A grain dealer bought 9,000 bushels of wheat, of which he sold 325 bu. to A, 640 bu. to B, 3,645 bu. to C, and 4,340 bu. to D. How many bushels remained unsold?

10. The firm of Davis & Brown has real estate worth \$3,100, machinery worth \$3,600, book accounts worth \$4,500, and cash on hand \$6,425. What are the total resources of the firm?

11. A merchant's gain for the first six months of the year was \$1,524, his total gain for the year was \$3,250. How much more did he gain during the last six months than during the first half of the year?

12. A speculator bought a house and lot for \$21,240, and sold it at a loss of \$960. What did he receive for it?

13. I sold a farm for \$9,324, which was \$1,300 more than it cost. What did I pay for the farm?

14. A gentleman left by will \$73,284 to his wife, son and daughter; \$17,380 to his daughter, \$20,000 to his son, and the remainder to his wife; how much did he will to his wife?

15. A mechanic's income for 1894 was as follows: Wages \$720, rent of house \$240, and interest on money in saving's bank \$36. His expenses were: meat bill \$95, baker's bill \$70, grocer's bill \$270, insurance and taxes \$64, clothing \$250, and other expenses \$84. How much did he save during the year?

16. A grocer's sales for the first half of the year were \$20,394; his sales for January were \$3,200; February, \$2,845; March, \$3,649; April, \$3,425; May, \$3,864. What were his sales for June?

17. During the five years' partnership, a firm gained \$250,400; if the gain for the first year was \$48,120, for the second, \$13,140, for the third as much as for the first and second, and for the fourth as much as for the second and third, less \$1,140; what was the gain for the fifth year?

18. I bought a lot for \$3,200; in building a house I paid for mason work \$1,825, for carpenter work \$6,240, for plumbing \$775, for plastering \$1,320, for painting \$725, for grading and sodding \$456, for interest on borrowed money to date of sale \$350; I then sold the property at a gain of \$125, receiving in exchange a house and lot worth \$2,500 and the balance in cash; how much cash did I receive?

19. A grain dealer bought the following: on Monday 964 bushels of wheat, 320 bushels of oats, 560 bushels of corn; on Tuesday 1,925 bushels of wheat, 364 bushels of oats, 725 bushels of corn; on Wednesday 3,640 bushels of wheat, 964 bushels of oats, 1,325 bushels of corn; on Thursday 2,164 bushels of wheat, 1,900 bushels of oats, 2,000 bushels of corn; on Friday 3,124 bushels of wheat, 1,624 bushels of oats, 984 bushels of corn; on Saturday 3,220 bushels of wheat, 984 bushels of oats, and 1,756 bushels of corn. What was the number of bushels of each kind of grain, and the total number of bushels of all kinds bought during the week?

MULTIPLICATION.

47. Multiplication is the process of finding the product of two numbers.

48. The **Multiplicand** is the number to be multiplied.

49. The **Multiplier** is the number to be multiplied by.

50. The **Product** is the result of the multiplication.

51. The **Sign of Multiplication** is \times , and means "multiplied by."

To the Teacher, and Student:—The first thing to be accomplished here is the thorough mastery of the multiplication table, at least as far as "twelve times twelve." It is *not* enough to be able to repeat the table in regular order; one must be able to "skip around" and announce instantly the result of any combination. Time spent here is time saved.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	368	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

PROOF OF MULTIPLICATION.

52. The Proof of Multiplication is similar to that already given for Addition, and will be found very short and practical.

ILLUSTRATIVE EXAMPLE.

$$326 \times 241 = 78566.$$

$$326 = 11 = 2.$$

$$241 = 7 \times 2 = 14 = 5.$$

$$\begin{array}{r} 326 \\ 1304 \\ 652 \\ \hline \end{array}$$

$$1304$$

$$652$$

$$78566 = 32, \text{ and } 3 + 2 = 5.$$

EXPLANATION.—The unite of the Multiplicand is 2; the unite of the Multiplier is 7, and $7 \times 2 = 14$, and the unite of 14 is 5. Now, find the unite of the product; this is found to be 5 also; which is proof of the correctness of the work.

Multiplication may also be proved by dividing the product by the multiplier; the quotient thus obtained should be the multiplicand; or the product divided by the multiplicand should give as a quotient, the multiplier.

GENERAL PROBLEMS.

1. What is the cost of 3,456 sheep at \$6 per head?
2. What is the cost of 1,964 horses at \$97 per head?
3. What is the cost of 3,574 cattle at \$28 per head?
4. What is the cost of 9,306 acres of land at \$36 per acre?
5. A man bought a farm containing 495 acres at \$28 per acre. What was the cost of the farm?
6. If a house is worth \$2,450, and the lot on which it stands is worth 6 times as much, what is the value of the lot?
7. What will 35 pianos cost at \$347 each?
8. What is the cost of 325 bushels of potatoes at 63 cents per bushel?
9. How much will 48 cannons cost at \$1,560 each?
10. What is the cost of 496 barrels of pork at \$17 per barrel?
11. A farmer sold 96 bushels of wheat at 57 cents a bushel. How much money did he receive?
12. What is the cost of 9 horses at \$212 each?

13. If a wagon cost \$65, a yoke of oxen 5 times as much, and a span of horses twice as much as the oxen; what was the cost of the horses?

14. What will 13 barrels of sugar, each containing 376 pounds, cost at 5 cents per pound?

15. What is the cost of 316 barrels of flour at \$5 per barrel?

16. If a train runs 49 miles in an hour, how far will it run in 48 hours?

CONTRACTIONS IN MULTIPLICATION.

53. Very many contractions in arithmetical processes are more ingenious than practical, and more time is required to learn them than is saved by their use. A few, however, are useful and these we give.

a. Where either factor contains ciphers.

ILLUSTRATIVE EXAMPLE.

Multiply 4600 by 74.

OPERATION.	
Right way.	Wrong way.
4600	4600
74	74
<hr/>	<hr/>
184	18400
322	32200
<hr/>	<hr/>
340400	340400

The writing of the ciphers in the partial products is altogether useless.

b. Where both factors contain ciphers.

ILLUSTRATIVE EXAMPLE.

Multiply 42000 by 2300.

OPERATION.
42000
2300
<hr/>
126
84
<hr/>
96600000

EXPLANATION.—Simply annex the ciphers in both factors to the integer product.

c. To multiply by 10, 100, 1000, 10000, etc.

Do not multiply at all, but merely annex the ciphers to the other factor. Thus, $746 \times 10000 = 7460000$.

d. Where one factor is very nearly 100, 1000, 10-000, etc.

ILLUSTRATIVE EXAMPLES.

Multiply 342 by 99.

OPERATION.

$$\begin{array}{r} 342 \times 100 = 34200 \\ 34200 - 342 = 33858 \text{ Ans.} \end{array}$$

EXPLANATION.—99 times 342 is once 342 less than 100 times 342.

Multiply 235 by 998.

OPERATION.

$$235000 - (2 \times 235) = 234530 \text{ Ans.}$$

EXPLANATION.— 235×998 is twice 235 less than 235×1000 ; hence $235000 - 470 = 235 \times 998$ or 234530.

e. Where one portion of the multiplier is a divisor of another portion.

ILLUSTRATIVE EXAMPLES.

Multiply 362 by 639.

OPERATION.

$$\begin{array}{r} 362 \\ 639 \\ \hline 3258 \times 7 \\ \hline = 22806 \\ 231318 \end{array}$$

EXPLANATION.—Multiplying 362 by 9 gives us our first partial product, 3258; since the remainder of the multiplier "63" is 7 times 9, multiply 3258 by 7 and we shall have 63 times 362 or 22806, and we thus save one multiplication, besides having to write but two partial products instead of three.

Study the following operations:—

$$\begin{array}{r} 523 \\ 287 \\ \hline 3661 (\times 4) \\ = 14644 \\ \hline 150101 \end{array}$$

$$\begin{array}{r} 2345 \\ 2408 \\ \hline 18760 (\times 3) \\ = 56280 \\ \hline 5646760 \end{array}$$

$$\begin{array}{r} 2536 \\ 756 \\ \hline 17752 (\times 8) \\ = 142016 \\ \hline 1917216 \end{array}$$

EXAMPLES FOR DRILL.

1. $76 \times 53000.$	8. $876 \times 99.$	15. $856 \times 357.$
2. $576000 \times 28.$	9. $54 \times 999.$	16. $328 \times 749.$
3. $32000 \times 460000.$	10. $763 \times 9999.$	17. $234 \times 4509.$
4. $758 \times 10000.$	11. $84 \times 98.$	18. $729 \times 369.$
5. $674 \times 2000.$	12. $236 \times 998.$	19. $267 \times 284.$
6. $506 \times 204000.$	13. $365 \times 549.$	20. $1604 \times 3612.$
7. $2700 \times 530.$	14. $743 \times 14412.$	21. $7186 \times 7209.$

DIVISION.

54. Division is the process of finding how many times one number contains another, also the process of separating numbers into equal parts.

55. The first of these processes is termed **Measurement Division**, the second, **Separative Division**.

56. The **Dividend** is the number to be divided.

57. The **Divisor** is the number by which we divide.

58. The **Quotient** is the result.

59. The **Remainder** is the undivided portion of the Dividend.

The true remainder is always less than the divisor.

60. The **Sign of Division** is \div , and means "divide by."

Division is also indicated by writing the Dividend over the Divisor with a horizontal line between them; thus, $\frac{24}{3}$, means that 24 is to be divided by 3.

GENERAL OBSERVATIONS.

61. Division is the basis of nearly all of the more complex operations of Arithmetic to which the student will be introduced later. The proper understanding of Fractions, Decimals, much of Denominate Numbers, Percentage, Proportion, and the Extraction of Roots, depends upon the thorough mastering of the principles and processes of Division. They are few in number, but broad in their application, and should be carefully committed.

PRINCIPLES.

1. *The Dividend equals the product of the Divisor and Quotient, plus the Remainder, if there be one.*
2. *The Dividend (less the Remainder), divided by the Quotient, equals the Divisor.*
3. *In Measurement Division, the Dividend and Divisor are like quantities, and the Quotient is abstract.*
4. *In Separative Division, the Dividend and Quotient are like quantities, while the Divisor is abstract.*
5. *The Remainder, being merely a part of the Dividend, is, of course, always like it in kind.*

RELATION OF THE QUOTIENT TO DIVIDEND
AND DIVISOR.

GENERAL PRINCIPLE.

(a.) *A change in the Dividend produces a corresponding change in the Quotient.*

ILLUSTRATIONS.

$\begin{array}{r} 48 \div 6 = 8 \\ 3 \qquad 3 \\ \hline 144 \div 6 = 24 \end{array}$	<p>EXPLANATION.—Multiplying the dividend, 48, by 3, gives 144, and this divided by the divisor, 6, gives us three times 8, or 24, for a new quotient.</p>
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Again :—

$\begin{array}{r} 2)48 \div 6 = 8 \\ \hline 24 \div 6 = 4 \end{array}$	<p>EXPLANATION.—We see here that dividing the dividend by 2 also divides the quotient by 2, provided that we keep the same divisor.</p>
--	---

(b.) *A change in the Divisor produces an opposite change in the Quotient.*

ILLUSTRATIONS.

$\begin{array}{r} 48 \div 6 = 8 \\ 2 \qquad 28 \\ \hline 48 \div 12 = 4 \end{array}$	<p>EXPLANATION.—Here, multiplying the divisor by 2 divides the quotient by 2.</p>
--	---

Also :—

$$48 \div 6 = 8$$

$$\begin{array}{r} 26 \\ 2 \end{array}$$

$$48 \div 3 = 16$$

EXPLANATION.—Here we see that dividing the divisor by 2, multiplies the quotient by 2.

(c.) *The same change in both Dividend and Divisor produces no effect upon the Quotient.*

ILLUSTRATIONS.

$$48 \div 6 = 8$$

$$\begin{array}{r} 2 \quad 2 \\ 96 \div 12 = 8 \end{array}$$

EXPLANATION.—Here, both dividend and divisor are multiplied by 2, and the quotient remains 8.

Also :—

$$2)48 \div 6 = 8$$

$$\begin{array}{r} 26 \\ 24 \div 3 = 8 \end{array}$$

EXPLANATION.—Here, both dividend and divisor are divided by 2, and the quotient is still 8.

Let the student compose many original illustrations of the above principles, until they are thoroughly fixed in his mind.

62. There are three general

FORMS OF DIVISION.

I. Short Division.—This form applies to those processes in which the divisor falls within the limits of the multiplication table. Thus, if the learner knows the multiplication table to 12×12 , he should use the Short Division process whenever the divisor does not exceed 12.

II. Long Division. This form applies to those processes in which the divisor is greater than any factor of the multiplication table. Students should not use Long Division where Short Division is possible.

III. Compound Division, or Cancellation.—This form is used in those cases where either Dividend or Divisor consists of two or more numbers joined by the sign of multiplication.

SHORT AND LONG DIVISION.

As the student of this book is supposed to be familiar with the ordinary processes of both Long and Short Division, we shall enter upon no exposition of these processes here.

SHORT PROCESSES.

1. When ciphers occur in the divisor strike them off, pointing off an equal number of figures from the right of the dividend. Then divide the remaining portion of the dividend by the remaining portion of the divisor, and annex to the remainder the figures of the dividend pointed off.

Thus, $24/00) 3462.13$ (144 Quotient.

$$\begin{array}{r}
 24 \\
 \hline
 106 \\
 96 \\
 \hline
 102 \\
 96 \\
 \hline
 613 \text{ Remainder.}
 \end{array}$$

2. When the divisor is 10, 100, 1000, etc., get the result by pointing off from the right of the dividend as many figures as there are ciphers in the divisor. The figures pointed off comprise the remainder, and the figures not pointed off comprise the quotient.

Thus: $296743 \div 10000 = 29.6743$. Quo. Rem.

63. To Prove Division employ Principle 1.

Thus: —

$$\begin{array}{r}
 7 \overline{) 56789} \\
 \underline{8112 \text{ Quo.; } 5 \text{ Rem.}} \\
 7 \\
 \underline{56784} \\
 5 \\
 \underline{56789} \text{ Dividend.}
 \end{array}$$

EXPLANATION.—By Principle 1, the quotient 8112, multiplied by the divisor 7, with the remainder 5 added should equal the dividend. As the operation shows this to be the case, the quotient and remainder are therefore correct.

EXAMPLES FOR DRILL.

(PROVE YOUR WORK.)

SHORT DIVISION.

- | | | |
|------------------------|-------------------------|----------------------------|
| 1. $2609 \div 3$. | 8. $41600213 \div 6$. | 15. $3016249 \div 4$. |
| 2. $102467 \div 5$. | 9. $4386012 \div 12$. | 16. $4671382 \div 9$. |
| 3. $78643 \div 7$. | 10. $1260134 \div 5$. | 17. $20176387 \div 50$. |
| 4. $258691 \div 6$. | 11. $3417201 \div 8$. | 18. $9671324 \div 700$. |
| 5. $365012 \div 8$. | 12. $41673861 \div 7$. | 19. $6938416 \div 9000$. |
| 6. $7496358 \div 11$. | 13. $4816701 \div 11$. | 20. $83765896 \div 1200$. |
| 7. $201658 \div 9$. | 14. $6571246 \div 12$. | 21. $3562340 \div 790$. |

LONG DIVISION.

- | | |
|--------------------------|-----------------------------------|
| 1. $178961 \div 13.$ | 11. $276401234 \div 3586.$ |
| 2. $297658 \div 19.$ | 12. $2763201786 \div 2761.$ |
| 3. $4026713 \div 39.$ | 13. $4387624 \div 450.$ |
| 4. $876593 \div 28.$ | 14. $201678358 \div 7800.$ |
| 5. $5400126 \div 54.$ | 15. $6900123 \div 680000.$ |
| 6. $7825893 \div 247.$ | 16. $234020360100 \div 15800000.$ |
| 7. $47136716 \div 326.$ | 17. $563020160 \div 5620000.$ |
| 8. $2867538 \div 285.$ | 18. $2486012016 \div 97640.$ |
| 9. $24178012 \div 327.$ | 19. $20010201020 \div 989600.$ |
| 10. $60217658 \div 601.$ | 20. $360172486 \div 856794.$ |

COMPOUND DIVISION.

ILLUSTRATIVE EXAMPLE.

Divide (29×5) by (8×5) .

OPERATION.

29×5	$\frac{29}{8} = 3 \frac{5}{8}$	Quo. Rem.	EXPLANATION.—Should we reduce the compound dividend and the compound divisor to single numbers by multiplying, we should have $\frac{29 \times 5}{8 \times 5} = \frac{145}{40}$. Now, by Gen. Prin. (c), we might divide our simple dividend and divisor by 5, and we should have $\frac{145}{40} = \frac{29}{8}$. Instead of adopting this longer process, we accomplish the same thing by simply striking out, or canceling the 5's, as in the operation.
8×5			

Hence :—

Striking out equal factors in the compound dividend and divisor is equal to dividing both dividend and divisor by the same number, and, therefore, does not affect the quotient.

ILLUSTRATIVE EXAMPLE.

Divide (52×6) by (12×5) .

OPERATION.

52×6	$\frac{52}{12} = 4 \frac{2}{3}$	Quo. Rem.	EXPLANATION.—Canceling the 6 in the dividend and the factor 6 in the 12, leaves 2, which we cancel together with the factor 2 in 52, leaving 26 in the dividend, this divided by the 5 gives the correct result.
12×5			

NOTE.—There are usually many ways in which the cancellation may be accomplished. The student should continue to cancel as long as there is any number that will divide any portion of both dividend and divisor, then combine the remaining factors in each and divide the combined dividend by the combined divisor.

EXAMPLES FOR DRILL.

Divide :—

1. $(1 \times 48 \times 50)$ by $(12 \times 5 \times 6)$.
2. $(96 \times 80 \times 125)$ by $(15 \times 16 \times 10)$.
3. $(1240 \times 81 \times 58 \times 76)$ by $(57 \times 87 \times 620)$.
4. $(8 \times 48 \times 56 \times 81 \times 52)$ by $(78 \times 27 \times 112 \times 32)$.
5. $(75 \times 156 \times 91 \times 68 \times 460)$ by $(26 \times 50 \times 85 \times 73)$.
6. $(95 \times 42 \times 54 \times 52 \times 90)$ by $(108 \times 18 \times 70 \times 38)$.
7. $(150 \times 81 \times 48 \times 91 \times 60)$ by $(128 \times 27 \times 135 \times 15)$.
8. $(175 \times 60 \times 256 \times 170)$ by $(425 \times 75 \times 64 \times 21)$.
9. $(49 \times 1957 \times 121 \times 84)$ by $(19 \times 343 \times 33 \times 133)$.
10. $(510 \times 650 \times 216 \times 910)$ by $(1000 \times 39 \times 26 \times 72)$.

APPLICATIONS OF DIVISION.

I. MEASUREMENT DIVISION.

64. Measurement Division is the process of finding how many times one number contains another of the same kind.

ILLUSTRATIVE PROBLEM.

At \$45 per acre, how much land can be bought for \$13,590?

FORM FOR LOGICAL ANALYSIS: \$45 is contained in \$13,590 302 times. Hence 302 acres may be bought.

NOTE.—Clear thinking and clear speaking are reciprocal. Let the student state the Logical Analysis for each problem.

GENERAL PROBLEMS.

1. At \$25 apiece, how many cows can be purchased with \$5000?
2. Allowing 65 trees for each row, how many rows of trees can be set out with 3528 trees?
3. If a man plows 275 sq. rds. per day, in how many days can he plow 1900 sq. rds.?

4. At \$1.25 per bushel, how many bushels of buckwheat can be bought with \$1750?

5. If a bicycle rider covers 24 ft. of ground per second, in how many seconds can he ride 1 mile of 5280 ft.?

6. Allowing 60 seconds per minute, how many minutes and seconds would he require to ride the mile?

7. If there are 231 cu. in. in 1 gal., how many gallons in 69748 cu. in.?

8. 160 sq. rds. make 1 acre. How many acres in 823000 sq. rds.?

9. If Battle Creek contains 17,896 people, and New York City 1,425,539, New York City is how many times larger than Battle Creek?

II. SEPARATIVE DIVISION.

65. Separative Division is the process of separating a number into equal parts.

PROBLEMS.

The cost of 17 horses was \$2465; what was the price of each horse?

LOGICAL ANALYSIS.—If \$2465 be divided into 17 equal parts, one part will be \$145, the price of each horse.

Or, $\frac{1}{17}$ of \$2465 equals \$145, the price of each horse.

NOTE.—In Separative Division, when there is a remainder, it should be written as a part of the Quotient with the Divisor under it, thus:—

$$\begin{array}{r} 9 \overline{) 248} \\ 27\frac{5}{9} \end{array}$$
 EXPLANATION.—The remainder is read as a part of the quotient, thus "twenty-seven and five-ninths." "Five-ninths" implies the unexecuted division of 5 by 9.

1. 9 heirs own an equal interest in a tract of land containing 41762 acres. What amount is owned by each heir?

2. A drove of cattle containing 263 head was sold for \$3900; find the average price of the cattle.

3. An orchard contains 21190 trees. If there are 65 rows, how many trees in each row?

4. \$10000 was expended by a contractor in moving 7860 cu. yds. of dirt; what was the average cost of removal per cubic yard?

NOTE.—To get the answer in dollars and cents, annex two ciphers to the whole cost before dividing.

5. A runner ran 3780 ft. in 96 seconds; how many feet did he run per second?

6. Mr. A paid \$16945 for 700 acres of land; find the cost per acre.

7. A string, 573 ft. long, is cut into 19 equal pieces; find the length of each piece.

8. John earned \$349.75 in 97 days; what did he earn per day?

9. Smith paid \$65 for the mowing of a field of grass containing 75 acres; find the cost per acre.

10. 16 men husked 75960 bu. of corn in 95 days; how many bushels were husked by each man per day?

PROBLEMS INVOLVING MEASUREMENT, AND
SEPARATIVE DIVISION.

NOTE.—Let the student state the Logical Analysis for each.

1. A farm of 316 acres was sold for \$7900; what was the price per acre?

2. At \$125 each, how many horses can be bought for \$4000?

3. A farmer harvested 3472 bu. of corn from a field containing 56 acres; what was the yield per acre?

4. A bank in New York City, during the year 1893, made loans to the amount of \$1,865,400, while a bank in Chicago, during the same year made loans to the amount of \$24,250,200; how many times greater were the loans of the Chicago bank than those of the New York City bank?

5. At 35 miles per hour, how many hours will be required for a train to run 1865 miles?

6. If potatoes yield 46 bu. per acre, how many acres will be required to yield 2500 bu.?

7. If there are 56 lbs. in a bushel of rye, how many bushels in a bin containing 128560 lbs.?

8. A farmer raised 26580 lbs. of oats from a field containing 37 acres; how many pounds were raised to the acre?

9. If a bushel of oats weighs 32 lbs., what was the yield per acre in the last problem?

10. If a bicycle rider rides 56 miles in a day, how long would it take him to ride around the earth, 25000 miles?

PROBLEMS INVOLVING COMPOUND DIVISION.

1. How many pounds of butter at 18 cts., should be given for 48 lbs. of sugar at 6 cts.?

2. Paid \$28 for 35 sacks of potatoes, each containing 96 lbs., what did I pay per pound for them?

3. When coffee is 6 lbs. for \$1, how much coffee should be given for 54 lbs. of butter at 12 cts.?

4. 16 gross of lead pencils, 144 in each gross, were given for 84 quires of paper, each containing 24 sheets; how many sheets of paper were given for each pencil?

5. A jobber sold a merchant 54 sacks of rice, each containing 98 lbs., and took in payment 18 bbls. of beef, averaging 196 lbs. per barrel, worth 12 cts. per pound; what was the rice worth per pound?

6. A tinner used 15 boxes of tin, each containing 72 sheets, each sheet 28 in. long and 24 in. wide, to make 45 crates of pans, each crate containing 12 doz. pans. How many square inches of tin were used to make one pan?

SUGGESTION.—The square inches in one sheet are found by multiplying the length by the breadth.

7. If 125 bundles of hides, each containing 60 lbs., were exchanged for 75 bbls. of oil, of 40 gals. each, worth 35 cts. per gallon; what were the hides worth per pound?

8. A farmer gave a nurseryman 35 cords of wood, worth \$7.50 per cord, for 15 bunches of apple trees, each bunch containing 125 trees; what did the farmer pay apiece for the trees?

9. At 15 lbs. for \$1, find the value of 105 lbs. of rice.

10. When sugar was selling at 16 lbs. for \$1, \$36 worth of sugar was given for 32 sacks of potatoes, each sack weighing 108 lbs. How many pounds of potatoes were given for 1 lb. of sugar?

GENERAL PROBLEMS.

NOTE.—When problems involve two or more steps in their solution, these should be indicated by equations, showing the several processes. We give an illustrative problem, with model solution, showing how this should be done.

Smith sold 567 bu. of wheat at 80 cts. He then purchased 17 hogs at \$3; 28 sheep at \$1.50; and paid his taxes amounting to \$42.95. How much cash had he remaining?

MODEL SOLUTION.

$$\begin{aligned}
 567 \times 80 \text{ cts.} &= \$453.60 \text{ received for wheat.} \\
 17 \times \$3 &= \$51 \text{ paid for hogs.} \\
 28 \times \$1.50 &= \$42 \text{ paid for sheep.} \\
 \$51 + \$42 + \$42.95 \text{ taxes} &= \$135.95 \text{ expended.} \\
 \$453.60 - \$135.95 &= \$317.65 \text{ Ans.}
 \end{aligned}$$

1. At 3 cts. per pound, find the total freight charges on 4 packages, weighing respectively 164 lbs., 246 lbs., 728 lbs., and 265 lbs.

2. John worked 196 days at \$1.75 per day. He paid during the time \$43.60 for clothes, \$19.75 for incidental expenses, and \$3.50 per week for board. What sum did he save?

3. A lady purchased 16 yds. of silk at \$1.80; 28 yds. of muslin at 15 cts.; 13 yards of dressgoods at 85 cts.; 7 collars at 35 cts., and gave the merchant a check for \$75. How much change should she receive?

4. A man bought a farm for \$2560; he stocked it with 18 head of horses, at an average cost of \$125; 65 head of cattle at \$26 each; 375 sheep at \$2.25 each. He then expended on improvements \$967, and sold the farm, including the stock, for \$5000; did he gain or lose, and how much?

5. If John can dig 65 lbs. of potatoes in 30 minutes, how many pounds could he dig in 5 hours?

6. A man had \$175.20 left after buying 65 cattle at \$45 per head, and 246 sheep at \$2.50 per head. What sum had he at first?

7. If A earns a salary of \$936 a year, how much does he earn in 75 days, allowing 26 working-days for each month?

8. If a man pays \$5.60 per week for board, what should his board bill be for 168 days?

9. A farmer sold 65 bu. of wheat at 75 cts.; 1265 lbs. of pork at 8 cts., and 946 lbs. of wool, receiving \$291.85 for the whole bill. What did he receive a pound for the wool?

10. If a man runs 8 rds. while a horse runs 29 rds., how far should a horse run while a man runs 72 rds.? How far should a man run while a horse runs 116 rds.?

11. If two men start from the same point and travel in opposite directions, one traveling 9 miles, and the other 13 miles per hour, how far apart would they be at the end of 47 hours?

12. In the above problem, how far apart would the men be, should they travel in the same direction for 24 hours?

13. At 75 cts. per yard, a dress pattern cost \$1.95 more than it would at 60 cts. per yard. How many yards of goods in the dress pattern?

14. A man bought a horse, saddle, and bridle; for the bridle he paid \$1.75, for the saddle \$13.50, and for the horse 5 times as much as for both saddle and bridle. What did all cost?

15. A finds that the cost of a cellar wall, including the laying of the stone, amounts to \$58.80, if he uses stone costing 67 cts. a perch; but if he uses stone costing 75 cts. a perch, the entire cost will be \$62.40. How many perch of stone were required for the wall, and what was the cost of laying?

16. Peck & Snyder bought 17 boxes of oranges at \$3.75 a box, each box containing 125 oranges. For what sum must they sell the whole lot in order to gain 2 cts. on each orange?

17. If, in the above problem, the oranges are retailed at 5 cts each, what sum will be gained allowing for 6 spoiled oranges to each box?

18. Bought 65946 lbs. of wool, at 19 cts. and sold it for 24 cts.; what sum was gained?

19. A huckster bought a lot of butter at 14 cts. per pound, and sold it for 17 cts.; he paid charges amounting to \$3.75 and then found that his net gain was \$20.40. How many pounds in the lot?

20. A contractor undertakes a job that requires the work of 18 men for 15 days to complete it. At the end of 5 days, 6 of the men quit work. How many additional men must be employed at the end of the 12th day, in order that the job may be completed within the contract time of 15 days?

FACTORING.

66. The Factors of a number are such numbers, as will produce the given number, if multiplied together. Thus, 6 and 5 are the factors of 30, and 2, 3, and 6 are the factors of 36.

67. A Prime Number is a number that cannot be factored, that is, one that is not divisible by any number except itself and unity, as 1, 3, 5, 7,—31.

68. A Composite Number is a number which can be factored. Thus, 15 is a Composite Number, because $3 \times 5 = 15$.

69. An Exact Divisor of a number is one which will divide it and leave no remainder.

NOTE.—Every exact divisor of a number may be a factor of that number

70. One number is said to be **divisible** by another when it contains that number an exact number of times.

DIVISIBILITY OF NUMBERS.

Two will divide any even number.

Three will divide any number the sum of whose digits is exactly divisible by 3.

Four will divide any number if the number expressed by its two right-hand figures is divisible by 4, or if they are ciphers.

Five will divide any number ending in 5 or 0.

Six will divide any even number the sum of whose digits is divisible by 3.

Eight will divide any number whose three right-hand figures are ciphers or if they express a number divisible by 8.

Nine will divide any number the sum of whose digits is divisible by 9.

Ten will divide any number ending in a cipher.

Twenty-five will divide any number if the number expressed by its two right-hand figures is divisible by 25.

71. To find the Prime Factors of a Composite Number.

ILLUSTRATIVE EXAMPLE.

Find the prime factors of 324.

SOLUTION.

$$\begin{array}{r} 2 \overline{) 324} \\ 2 \overline{) 162} \\ 3 \overline{) 81} \\ 3 \overline{) 27} \\ 3 \overline{) 9} \\ 3 \end{array}$$

EXPLANATION.—Since 324 is an even number, divide it by 2, a prime number, obtaining 162; then 162, being an even number, divide it by 2, obtaining 81; since the sum of the figures of 81 ($8 + 1 = 9$) is divisible by 3, a prime number, take 3 as the next divisor and the quotient is 27; again take 3 as a divisor and the quotient is 9; again divide by 3 and the quotient is 3, a prime number. Hence, the prime factors of 324 are the divisors 2, 2, 3, 3, 3, and the quotient 3.

72. RULE.—*Divide the given number by its smallest prime factor, then divide the resulting quotient in the same manner, until a prime number is obtained as a quotient. The several divisors and the last quotient will be the prime factors.*

73. PROOF.—The continued product of all the prime factors should equal the given number.

74. The Exact Divisors of a number consist, (a) of all of its prime factors; (b) of the numbers obtained by multiplying any two or more of the prime factors together.

EXAMPLES FOR DRILL.

Find the prime factors of —

1. 36	3. 560	5. 312	7. 34850	9. 353430
2. 144	4. 924	6. 1245	8. 45625	10. 534303

11. Write all of the exact divisors of 27, 60, 54, 72, 96.

FACTORING BY INSPECTION.

75. It is indispensable to the expert calculator to be able to factor at sight, all numbers less than 100, also, to produce any of these numbers from their factors, and to recognize at sight any prime number less than 100.

To be able to do this, requires but a few days practice, and the advantage gained will amply repay the student for the time so expended.

There are many combinations constantly arising, not found in the multiplication table, but the result of which the ready arithmetician should be able to announce instantly. Such combinations as, 5×13 , 3×17 , 5×19 , 2×27 , 4×15 , 6×13 , etc. should be thoroughly committed; also the ability should be acquired to separate instantly such numbers as, 51, 76, 52, 85, 39, etc. into their factors. Suppose it is necessary to perform a compound division like the following:

$\frac{95 \times 51 \times 129}{34 \times 76}$. It saves much time to strike out the factor 19 from

95 and 76, and the factor 17 from 51 and 34. The problem is then reduced to $\frac{5 \times 3 \times 129}{2 \times 4}$, a great saving of work. The undrilled stu-

dent would probably observe no opportunity for cancellation in this problem, yet such combinations are constantly occurring in Fractions, Percentage, Proportion, and all of the many applications of Compound Division.

We urgently advise the complete mastery of the following tables, also thorough drill in the exercises that follow. Much of the inefficiency so often shown by students in the subject of Fractions and Percentage is caused by unfamiliarity with the properties of ordinary numbers.

FACTOR TABLE.

26 = 2×13	56 = $\begin{cases} 2 \times 28 \\ 4 \times 14 \end{cases}$	72 = $\begin{cases} 2 \times 36 \\ 4 \times 18 \\ 3 \times 24 \end{cases}$	88 = $\begin{cases} 2 \times 44 \\ 4 \times 22 \end{cases}$
28 = 2×14	57 = 3×19	74 = 2×37	90 = $\begin{cases} 2 \times 45 \\ 3 \times 30 \\ 5 \times 18 \\ 6 \times 15 \end{cases}$
32 = 2×16	58 = 2×29	75 = $\begin{cases} 3 \times 25 \\ 5 \times 15 \end{cases}$	91 = 7×13
36 = 2×18	60 = 4×15	76 = $\begin{cases} 2 \times 38 \\ 4 \times 19 \end{cases}$	92 = $\begin{cases} 2 \times 46 \\ 4 \times 23 \end{cases}$
38 = 2×19	62 = 2×31	78 = $\begin{cases} 2 \times 39 \\ 3 \times 26 \\ 6 \times 13 \end{cases}$	93 = 3×31
39 = 3×13	63 = 3×21	81 = 3×27	94 = 2×47
42 = $\begin{cases} 2 \times 21 \\ 3 \times 14 \end{cases}$	64 = $\begin{cases} 2 \times 32 \\ 4 \times 16 \end{cases}$	82 = 2×41	95 = 5×19
45 = 3×15	65 = 5×13	84 = $\begin{cases} 2 \times 42 \\ 3 \times 28 \\ 6 \times 14 \end{cases}$	96 = $\begin{cases} 2 \times 48 \\ 3 \times 32 \\ 4 \times 24 \\ 6 \times 16 \end{cases}$
46 = 2×23	66 = $\begin{cases} 3 \times 22 \\ 2 \times 33 \end{cases}$	85 = 5×17	98 = $\begin{cases} 2 \times 49 \\ 7 \times 14 \end{cases}$
48 = $\begin{cases} 2 \times 24 \\ 3 \times 16 \end{cases}$	68 = $\begin{cases} 2 \times 34 \\ 4 \times 17 \end{cases}$	86 = 2×43	99 = 3×33
51 = 3×17	69 = 3×23	87 = 3×29	
52 = $\begin{cases} 2 \times 26 \\ 4 \times 13 \end{cases}$	70 = $\begin{cases} 2 \times 35 \\ 5 \times 14 \end{cases}$		
54 = $\begin{cases} 2 \times 27 \\ 3 \times 18 \end{cases}$			

TABLE OF PRIME NUMBERS.

1, 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

DEFINITIONS.

76. A Power is a number that may be separated into two or more equal factors. Thus, $25 = (5 \times 5)$, $8 = (2 \times 2 \times 2)$, $144 = (12 \times 12)$, and $81 = (3 \times 3 \times 3 \times 3)$ are powers.

77. The Root of a power is the factor repeated in producing the power. Thus, 3 is the 2nd or "square" root of 9, 2 is the 3rd or "cube" root of 8, 12 is the 2nd root of 144, 3 is the 4th root of 81, etc.

78. A power is often indicated by affixing a small figure called an **Exponent** to the root; thus $2^2 =$ the 2nd power of 2, $= 4$. $5^3 =$ the 3rd power of 5 $= 125$, etc.

NOTE.—The first power of a number is the number itself as, $5^1 = 5$, etc.

It is important to know at sight the lower powers of the smaller prime numbers. Learn thoroughly the following

TABLE OF POWERS.

$2^2 = 4$	$2^4 = 16$	$2^6 = 64$	$3^3 = 27$	$5^2 = 25$	$7^2 = 49$
$2^3 = 8$	$2^5 = 32$	$3^2 = 9$	$3^4 = 81$	$5^3 = 125$	

DRILL EXERCISES.

NOTE.—The questions refer to numbers within the limits of the tables, that is, to numbers less than 100.

1. Separate each of the following numbers into as many pairs of factors as possible. 78, 96, 24, 72, 98, 84.

2. What numbers contain 13? 17? 19? 23? 29?

3. What is the largest prime number that will divide all of the numbers in each of the following combinations:—

a. 91, 78, 65, 52. b. 85, 68, 51. c. 76, 95, 57. d. 58, 87.

4. What numbers contain 14? 15? 18? 24?

5. What number contains both 12 and 16? 24 and 9? 15, 20, and 12? 9, 24, and 36? 14, 12, and 21?

6. What is the highest power (h. p.) of 2 in each of the following numbers? 68, 48, 72, 96, 50, 76, 40.

Exp.—The highest power of 2 in 68 is 4; in 48, is 16, etc.

7. What is the h. p. of 2 in 56? of 3 in 72? of 5 in 250? of 7 in 98? of 3 in 108?

8. What is the h. p. of 2 that is contained in *all* of the following numbers: 24, 48, 72, 88?

9. Factor mentally the following numbers: 54, 96, 60, 75, 72. Which of them contains the h. p. of 2? of 3? of 5? of 7?

10. Reduce the following compound division to its simplest form by striking out all factors common to both dividend and divisor.

$$\frac{76 \times 51 \times 96 \times 91 \times 98 \times 75 \times 50}{72 \times 52 \times 70 \times 125 \times 57 \times 85}$$

GREATEST COMMON DIVISOR.

79. A Common Divisor of two or more numbers is any number that will exactly divide each of them. Thus, 6 is a common divisor of 12, 18, and 30.

80. The Greatest Common Divisor (G. C. D.) of two or more numbers is the largest number that will exactly divide each of them. Thus, 12 is the G. C. D. of 24, 36, and 48.

PRINCIPLES.

1. *The G. C. D. of two or more numbers is the product of the highest powers of the prime factors found in all the numbers.*

2. *A Common Divisor of two or more numbers is a divisor of their sum, their difference, and of the remainder after division.*

81. There are two Methods employed for finding the G. C. D., either of which may be used, according to the size of the numbers involved; viz. the Factoring Methods, and the Division Method.

FACTORING METHOD.

82. This Method is based upon Principle 1

ILLUSTRATIVE EXAMPLE.

Find the G. C. D. of 72, 48, 108, 96.

OPERATION.

$$72 = 2^3 \times 3^2$$

$$144 = 2^4 \times 3^2$$

$$108 = 2^2 \times 3^3$$

$$180 = 2^2 \times 3^2 \times 5$$

$$2^2 \times 3^2 = 36 \text{ G. C. D.}$$

EXPLANATION.— 2^2 or 4 is the h. p. of 2 found in *all* the numbers, also 3^2 or 9 is the h. p. of 3 found in *all* of the numbers, hence 4×9 or 36 = the G. C. D.

83. A more convenient application of this method is the following:—

OPERATION.					
2	72	144	108	180	
2	36	72	54	90	
3	18	36	27	45	
3	6	12	9	15	
	2	4	3	5	

divisor, the product of the divisors used, $2 \times 2 \times 3 \times 3 = 36$, is the G. C. D.

84. In practical Arithmetic it is seldom necessary to find the G. C. D. of numbers greater than 100, so that if the tables under Art. 75 have been mastered, the G. C. D. can at once be announced by inspection. Thus it is easy to see that 24 is the G. C. D. of 48, 72, and 96; that 21 is the G. C. D. of 42, 63, and 84; or that 13 is the G. C. D. of 52, 65, and 91, etc.

DIVISION METHOD.

NOTE.—This method applies to large numbers, and although seldom used, is here given.

ILLUSTRATIVE EXAMPLE.

Find the G. C. D. of 1056, 672, 264.

OPERATION.

$$\begin{array}{r}
 672 \overline{) 1056} \quad (1 \\
 \underline{672} \\
 384 \overline{) 672} \quad (1 \\
 \underline{384} \\
 288 \overline{) 384} \quad (1 \\
 \underline{288} \\
 96 \overline{) 288} \quad (3 \\
 \underline{288}
 \end{array}$$

$$\begin{array}{r}
 96 \overline{) 264} \quad (2 \\
 \underline{192} \\
 72 \overline{) 96} \quad (1 \\
 \underline{72} \\
 24 \overline{) 72} \quad (3 \\
 \underline{72}
 \end{array}$$

EXPLANATION.—Applying Prin. 2, we divide any one of the numbers by any other one, as 1056 by 672. The remainder resulting from this division, contains the G. C. D. if there be one; so we can use this remainder as a new divisor, dividing it into the first divisor, 672, and so on, until we find an exact divisor, 96, which is the

G. C. D. of the numbers 1056 and 672. Now performing the same operation with this exact divisor, and the third number 264, we find the G. C. D. of the three numbers to be 24.

EXAMPLES FOR DRILL.

Find the G. C. D. of—

- FACTORING METHOD.
1. 96, 144, 192, 480.
 2. 144, 216, 504, 576, 720.
 3. 196, 280, 224, 308, 560.

- DIVISION METHOD.
4. 468, 572.
 5. 703, 1517.
 6. 203, 319, 232.

BY INSPECTION.

7. 63, 84, 42.

8. 64, 96, 72.

9. 45, 90, 75.

10. 36, 90, 54, 72.

LEAST COMMON MULTIPLE.

85. A Multiple of a Number is any number that will exactly contain that number. Thus, 51 is a multiple of 17. 60 is a multiple of 12, etc.

86. A Common Multiple of two or more numbers is any number that will exactly contain each of them. Thus, 96 is a common multiple of 4, 16, and 12; 75 is a common multiple of 15 and 3.

87. The Least Common Multiple (L. C. M.) of two or more numbers is the least number that will exactly contain each of them. Thus, 75 is the L. C. M. of 15 and 25; 96 is the L. C. M. of 24, 32, and 16.

PRINCIPLES.

1. *A multiple of a number is also a multiple of any of the exact divisors of that number.*

2. CONVERSELY.—*If a given number contains all the exact divisors of another number, the given number will be a multiple of that number.* Thus, 2, 3, 4, 6, 8, and 12 are all of the exact divisors of 24. Now the number 168 exactly contains these divisors and therefore is a multiple of 24.

3. *The L. C. M. of two or more numbers is equal to the product of the highest powers of the factors found in any of these numbers.*

88. There are three methods of finding the L. C. M. of numbers, each based on Prin. 3. We give each.

FACTORING METHOD.

ILLUSTRATIVE EXAMPLE.

What is the L. C. M. of 80, 108, 75, 24, and 60?

OPERATION.

$$80 = 2^4 \times 5$$

$$108 = 2^2 \times 3^3$$

$$75 = 3 \times 5^2$$

$$24 = 2^3 \times 3$$

$$60 = 2^2 \times 3 \times 5$$

$$2^4 \times 3^3 \times 5^2 = 16 \times 27 \times 25 = 10800 \text{ L. C. M.}$$

EXPLANATION.—The highest power of 2, found in any of these numbers, is $2^4 = 16$, which is found in 80. The highest power of 3 is 3^3 or 27, found in 108, and the highest power of 5 is 5^2 , or 25, found in 75. As there are no powers of other factors in any of the numbers, the L. C. M. will therefore be $16 \times 27 \times 25$, or 10800. By thus selecting and combining the *highest powers*, it is evident that we shall get a number that will contain all the *lower powers*, and therefore (Prin. 2) a number that will contain all the given numbers.

DIVISION METHOD.

89. When dealing with large numbers, we may employ a more convenient method, similar to the one used in Art. 86, for obtaining the G. C. D.

OPERATION.

2	80	108	75	24	60
2	40	54	75	12	30
2	20	27	75	6	15
3	10	27	75	3	15
5	10	9	25	1	5
	2	9	5	1	1

$$2 \times 2 \times 2 \times 3 \times 5 \times 2 \times 9 \times 5 = 10800 \text{ L. C. M.}$$

EXPLANATION.—Divide the given numbers by any prime number that will exactly divide two or more of them; bring down such numbers as are not thus divisible. Continue dividing until there is no prime number that will exactly divide any two of the remaining numbers. The product of the several divisors and the several remaining numbers will be the L. C. M. It is better to begin with the lowest factors and exhaust the divisions with that one before taking another. It will be seen that the resulting numbers comprise the same factor-powers as those selected in forming the L. C. M. by the first method.

INSPECTION METHOD.

90. In the operations of practical arithmetic it is seldom necessary to find the L. C. M. of numbers greater than 100. In such numbers the factors comprising the L. C. M. may be easily obtained by inspection, provided, of course, that the student is familiar with the number-tables in Art. 75.

ILLUSTRATIVE EXAMPLE.

What is the L. C. M. of 72, 60, 28, 48, 75?

OPERATION.

$$16 \times 9 \times 25 \times 7 = 25200 \text{ L. C. M.}$$

EXPLANATION.—The required L. C. M., (Prin. 3) equals the product of the highest powers of the prime factors found in any of these

numbers. By inspection, we see that 16, a factor of 48, is the h. p. of 2; that 9, a factor of 72, is the h. p. of 3; that 25, a factor of 75, is the h. p. of 5; that 7, a factor of 28, is the h. p. of 7; and that there are no other prime factors involved in the numbers. The L. C. M. is therefore $16 \times 9 \times 25 \times 7$, or 25200.

We earnestly advise the student to use this method wherever possible, as it is a great time-saver.

EXAMPLES FOR DRILL.

(SELECT THE BEST METHOD.)

NOTE.—For convenience, omit such of the smaller numbers as are factors of any of the larger ones; their omission will, of course, make no difference in the result.

Find the L. C. M. of —

- | | |
|------------------------------------|---------------------------------|
| 1. 24, 50, 60, 56, 90. | 6. 144, 288, 90, 45, 160. |
| 2. 240, 160, 120, 144, 72, 36, 12. | 7. 51, 85, 56, 24, 34. |
| 3. 48, 60, 54, 45, 75. | 8. 100, 54, 75, 60, 28. |
| 4. 26, 65, 18, 39, 91. | 9. 250, 175, 48, 36, 125. |
| 5. 56, 98, 70, 28, 35. | 10. 12, 30, 63, 28, 42, 84, 90. |

COMMON FRACTIONS.

91. A Fraction is the expression of one or more of the equal parts of a quantity. Its form is that of an unexecuted division. (See Note, Art. 65.)

92. The Unit, or Base, of a fraction, is the thing, or quantity, to which the fraction applies. Thus, in the fraction $\frac{5}{7}$, the abstract unit 1 is the base; in the expression " $\frac{5}{6}$ of an acre," an acre is the base; in the expression " $\frac{3}{5}$ of \$12," \$12 is the base; in the expression " $\frac{2}{3}$ of $\frac{3}{4}$," $\frac{3}{4}$ is the base of the fraction $\frac{2}{3}$. In expressions like the last, the fraction with its base is called a **Compound Fraction**.

93. The Terms of a fraction are the figures used in expressing it.

94. The Denominator is the term written below the line, and names the equal parts into which the base of the fraction is divided.

95. The Numerator is the term written above the line, and states the number of parts taken. Thus, in the expression " $\frac{2}{3}$ of an apple," the 3 denominates, or names the parts as "thirds," while the numerator, 2, implies, that two of these parts are taken.

96. In order to fix these terms and relations *firmly in the mind*, let the pupil analyze the following fractions, in accordance with the models given.

ILLUSTRATIVE EXAMPLE.

Analyze: $\frac{5}{7}$ of an apple.

MODEL.—" $\frac{5}{7}$ of an apple" means that the base, an apple, has been divided into 7 equal parts, and five of these parts are taken.

Analyze:—

- | | |
|--|---|
| 1. $\frac{3}{8}$ of a pound. | 6. $\frac{4}{5}$ of \$7. |
| 2. $\frac{7}{9}$ of a dollar. | 7. $\frac{5}{6}$ of John's money. |
| 3. $\frac{11}{13}$ of a month's wages. | 8. $\frac{3}{4}$ of 21 yds. of carpet. |
| 4. $\frac{3}{10}$ of a gallon. | 9. $\frac{5}{9}$ of $\frac{2}{3}$ bushels. |
| 5. $\frac{15}{19}$ of a yard. | 10. $\frac{16}{23}$ of the value of a ship. |

97. A Common Fraction is one whose denominator is not a power of 10. Thus, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{9}{14}$, $\frac{7}{20}$, etc. are common fractions.

98. A Decimal Fraction is one whose denominator is a power of 10. Thus, $\frac{5}{10}$, $\frac{7}{100}$, $\frac{16}{1000}$, etc. are decimal fractions.

99. A Proper Fraction is one whose numerator is less than its denominator, as, $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{19}$.

100. An Improper Fraction is one whose numerator is equal to, or greater than, its denominator, as $\frac{4}{4}$, $\frac{6}{5}$, $\frac{7}{7}$, $\frac{9}{8}$.

101. A Mixed Number consists of an integer and a fraction united into one expression, as, $2\frac{3}{4}$, $5\frac{1}{3}$, $17\frac{1}{2}$.

GENERAL OBSERVATION.

102. As has already been shown (Art. 65, Note), a fraction is an expression of the result of an unexecuted division, thus, $4 \div 5 = \frac{4}{5}$,

$9 \div 2 = \frac{9}{2}$, etc. From this it appears that the numerator and denominator of a fraction constitute, respectively, a dividend and divisor, and that the fraction itself represents the quotient of this division. It is therefore evident that the value of a fraction depends upon the same general law that regulates the value of a quotient in an ordinary division. This, as applied to a fraction, is best expressed thus :—

PRINCIPLE.

(a.) *Multiplying the numerator, or dividing the denominator, multiplies the fraction.*

(b.) *Dividing the numerator, or multiplying the denominator, divides the fraction.*

(c.) *Multiplying or dividing both terms of a fraction by the same number does not affect the value of the fraction.*

Apply the above principle to the following :—

DRILL EXERCISES.

1. Multiply $\frac{5}{8}$ by 2, in two ways.
2. Divide $\frac{12}{9}$ by 3, in two ways.
3. Change $\frac{5}{6}$ to a fraction of equal value, by multiplying both terms by 4.
4. Change $\frac{12}{8}$ to an equivalent fraction, by dividing both terms by 6.
5. Find two fractions, each equal to $\frac{3}{4}$.
6. Change $\frac{8}{12}$ to two equivalent fractions, one formed by division, and one by multiplication.
7. Multiply $\frac{7}{18}$ by 9, giving two answers.
8. Divide $\frac{24}{5}$ by 6, giving two answers.
9. Change $\frac{16}{4}$ to an equivalent fraction whose terms are smaller numbers.
10. Change $\frac{5}{7}$ to an equivalent fraction whose terms are larger numbers.

REDUCTION OF FRACTIONS.

103. Reduction of Fractions is the process of changing their form without altering their value.

104. There are two general kinds of reduction.

I. REDUCTION TO LOWER TERMS.

105. This kind of reduction is accomplished by *dividing both terms of the fraction by the same number*, as in example 9 of the preceding exercise.

106. To reduce a fraction to **Lowest Terms**, divide both terms by their G. C. D.

ILLUSTRATIVE EXAMPLE.

Reduce $\frac{48}{72}$ to lowest terms.

OPERATION.

By inspection the G. C. D. of 48 and 72 is found to be 24.

$$\frac{48 \div 24}{72 \div 24} = \frac{2}{3} \text{ Ans.}$$

NOTE.—We may reach the same result by dividing both the terms by any common divisor, and continuing until the terms are prime to each other; thus, $\frac{48 \div 2}{72 \div 2} = \frac{24 \div 2}{36 \div 2} = \frac{12 \div 6}{18 \div 6} = \frac{2}{3}$. The larger the common divisor used, the shorter will be the process.

EXAMPLES FOR DRILL.

Reduce the following fractions to their lowest terms:—

$$1. \frac{25}{85} \mid 2. \frac{19}{38} \mid 3. \frac{44}{121} \mid 4. \frac{1640}{3240} \mid 5. \frac{6}{80} \mid 6. \frac{45}{60} \mid 7. \frac{39}{117} \mid 8. \frac{340}{1728} \mid 9. \frac{68}{85} \mid 10. \frac{570}{950}$$

II. REDUCTION TO HIGHER TERMS.

107. This kind of reduction is accomplished by *multiplying both terms of the fraction by the same number*, as in Ex. 10, Art. 102.

108. There are two general applications of this operation.

REDUCING A FRACTION TO A GIVEN DENOMINATOR.

ILLUSTRATIVE EXAMPLE.

Reduce $\frac{5}{8}$ to 24ths.

OPERATION.

$$24 \div 8 = 3$$

$$\frac{5}{8} \times 3 = \frac{15}{24}$$

$$\frac{8}{8} \times 3 = \frac{24}{24}$$

Ans.

EXPLANATION.—Since $\frac{5}{8}$ is to be changed to a fraction whose denominator is 24, we first divide 24 by 8, in order to find the number, 3, by which we must multiply both terms of the fraction in order to effect the reduction.

Then multiplying both terms of $\frac{5}{8}$ by 3 gives us $\frac{15}{24}$, the required fraction.

NOTE.—For purposes of reduction, it is sometimes convenient to express whole numbers in the fractional form, by writing the whole number as the numerator of a fraction, with 1 for its denominator, thus, $5 = \frac{5}{1}$, $16 = \frac{16}{1}$, etc.

EXAMPLES FOR DRILL.

- | | |
|---|---------------------------|
| 1. Reduce $\frac{3}{8}$ of a yard to 16ths. | 6. Reduce 36 to 15ths. |
| 2. Reduce $\frac{4}{7}$ of a pound to 35ths. | 7. Reduce 28 to 9ths. |
| 3. Reduce $\frac{1}{3}$ of a foot to 27ths. | 8. Reduce 1,348 to 24ths. |
| 4. Reduce $\frac{5}{24}$ of an ounce to 144ths. | 9. Reduce 2,439 to 79ths. |
| 5. Reduce 9 to thirds. | 10. Reduce 794 to 13ths. |

REDUCING FRACTIONS TO A COMMON DENOMINATOR.

109. Fractions are said to have a **Common Denominator** when their denominators are alike, as $\frac{7}{12}$, $\frac{8}{12}$, $\frac{5}{12}$.

110. The required denominator is always a **Multiple** of the denominator of the given fraction.

111. Any series of fractions having **Different Denominators**, may be reduced to another series, having a **Common Denominator**. Thus, the fractions $\frac{5}{8}$, $\frac{2}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, may be reduced to the series $\frac{15}{24}$, $\frac{16}{24}$, $\frac{18}{24}$, $\frac{20}{24}$.

PRINCIPLES.

1. The *Common Denominator* is a common multiple of the several denominators of the given fractions.
2. The *Least Common Denominator* (L. C. D.) is the L. C. M. of the several denominators of the given fractions.

NOTE.—The latter of these principles only, is used in practice.

ILLUSTRATIVE EXAMPLE.

Reduce $\frac{5}{6}$, $\frac{7}{12}$, $\frac{9}{16}$, $\frac{5}{24}$, to L. C. D.

EXPLANATION.—The L. C. M. of 6, 12, 16, and 24, is 48. Then by Art. 108. $\frac{5}{6} = \frac{40}{48}$, $\frac{7}{12} = \frac{28}{48}$, $\frac{9}{16} = \frac{27}{48}$, and $\frac{5}{24} = \frac{10}{48}$, or in series, $\frac{5}{6}$, $\frac{7}{12}$, $\frac{9}{16}$, $\frac{5}{24} = \frac{40}{48}$, $\frac{28}{48}$, $\frac{27}{48}$, $\frac{10}{48}$.

112. RULE.—Find the L. C. M. of the several denominators of the given fractions; then change each fraction to an equivalent fraction having this L. C. M. for a denominator, as in Art. 108.

EXAMPLES FOR DRILL.

Reduce the following fractions to equivalent fractions having the L. C. D.

NOTE.—Find the L. C. D. by inspection, as suggested in Art. 90.

- | | | |
|--|-----|--|
| 1. $\frac{3}{4}$, $\frac{1}{8}$, $\frac{1}{2}$. | 7. | $\frac{4}{15}$, $2\frac{1}{2}$, $\frac{3}{4}$, $\frac{5}{6}$. |
| 2. $\frac{3}{8}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{1}{5}$. | 8. | 9, $2\frac{1}{4}$, $\frac{1}{2}$, $3\frac{2}{3}$, $6\frac{1}{2}$. |
| 3. $\frac{5}{6}$, $\frac{7}{12}$, $\frac{14}{15}$. | 9. | $\frac{3}{10}$, $\frac{1}{4}$, $\frac{13}{18}$, 7, $\frac{5}{9}$, $16\frac{1}{2}$. |
| 4. $\frac{7}{8}$, $\frac{3}{16}$, $\frac{17}{20}$. | 10. | $4\frac{5}{9}$, $\frac{17}{18}$, $\frac{6}{5}$, $\frac{19}{20}$, 3, $4\frac{1}{2}$. |
| 5. $\frac{4}{5}$, $\frac{7}{8}$, $\frac{2}{7}$, $\frac{1}{2}$. | | |

REDUCING MIXED NUMBERS TO IMPROPER FRACTIONS.

ILLUSTRATIVE EXAMPLE.

Reduce $23\frac{2}{3}$ to an improper fraction.

SOLUTION.— $23 \times \frac{3}{3} = \frac{69}{3} + \frac{2}{3} = \frac{71}{3}$. Ans.

EXPLANATION.—Since 1 unit is equal to 3 thirds, 23 units, or 23 times 1 unit, are equal to 23 times 3 thirds, or $\frac{69}{3} + \frac{2}{3} = \frac{71}{3}$; therefore, $23\frac{2}{3} = \frac{71}{3}$.

113. RULE.—Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the sum over the denominator.

Reduce to improper fractions:—

- | | |
|----------------------------|------------------------------|
| 1. $3\frac{1}{2}$ gallons. | 6. $23\frac{7}{8}$ cords. |
| 2. $2\frac{1}{4}$ feet. | 7. $10\frac{13}{18}$ miles. |
| 3. $5\frac{1}{3}$ inches. | 8. $38\frac{3}{11}$ barrels. |
| 4. $6\frac{3}{8}$ miles. | 9. $516\frac{3}{8}$ bushels. |
| 5. $16\frac{1}{4}$ feet. | 10. $3948\frac{5}{8}$ acres. |

REDUCING IMPROPER FRACTIONS TO WHOLE OR MIXED NUMBERS.

ILLUSTRATIVE EXAMPLE.

Reduce $\$2\frac{3}{8}$ to a whole or mixed number.

SOLUTION. **EXPLANATION.**—Since there are 8 eighths in 1 whole dollar, there must be as many whole dollars in $\frac{23}{8}$ as 8 is contained times in 23, or 2 whole dollars and $\frac{7}{8}$ remaining; therefore, $\$2\frac{3}{8} = \$2\frac{7}{8}$.

EXAMPLES FOR DRILL.

Reduce the following to integers or mixed numbers:—

- | | |
|---------------------------------------|----------------------------------|
| 1. $\frac{11}{2}$ pounds. | 6. $\frac{364}{5}$ barrels. |
| 2. $\frac{11\frac{1}{2}}{4}$ gallons. | 7. $\frac{33640}{25}$ yards. |
| 3. $\frac{25}{8}$ yards. | 8. $\frac{9160}{216}$ hours. |
| 4. $\frac{33}{8}$ bushels. | 9. $\frac{13124}{144}$ barrels. |
| 5. $\frac{192}{8}$ feet. | 10. $\frac{13169}{325}$ gallons. |

ADDITION OF FRACTIONS.

114. Addition of Fractions is the process of finding the sum of two or more fractional numbers.

115. Before being added or subtracted, fractions must have a Common Denominator and apply to the same base, or unit.

ILLUSTRATIVE EXAMPLES.

Find the sum of $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{1}{2}$.

MODEL SOLUTION.

$$\left. \begin{array}{l} \frac{5}{6} = 10 \\ \frac{3}{4} = 9 \\ \frac{2}{3} = 8 \\ \frac{1}{2} = 6 \end{array} \right\} 12 \text{ L. C. D.}$$

EXPLANATION.—The L. C. D. of the fractions is 12. Reducing each fraction to this denominator, we have the several new numerators, 10, 9, 8, and 6. Adding these, we find the sum of the several fractions to be $\frac{33}{12}$, which, reduced to a mixed number in its lowest terms, is $2\frac{3}{4}$, the required sum.

$$\frac{33}{12} = 2\frac{9}{12} = 2\frac{3}{4} \text{ Ans.}$$

Find the sum of $6\frac{1}{8}$, $5\frac{2}{3}$, $\frac{7}{12}$, $13\frac{1}{6}$, $7\frac{9}{16}$

MODEL SOLUTION.

$$\left. \begin{array}{l} 6 \left| \frac{1}{8} = 6 \right. \\ 5 \left| \frac{2}{3} = 32 \right. \\ \frac{7}{12} = 28 \\ 13 \left| \frac{1}{6} = 8 \right. \\ 7 \left| \frac{9}{16} = 27 \right. \end{array} \right\} 48 \text{ L. C. D.}$$

$$31 \quad \frac{101}{48} = 2\frac{5}{48}$$

$$\frac{31}{33\frac{5}{48}} \text{ Ans.}$$

EXPLANATION.—Adding the fractional parts of the quantities, we find by the process employed in the preceding example, that their sum is $2\frac{5}{48}$. Adding the whole numbers separately, we find their sum to be 31, which added to $2\frac{5}{48}$, the sum of the fractions, gives $33\frac{5}{48}$, the entire sum.

NOTE.—In business, instead of writing $13\frac{1}{6}$ yards, $6\frac{1}{2}$ yards, or $3\frac{1}{2}$ yards, they are written as 13¹ yards, 6² yards, 3² yards, omitting the denominators.

Ounces are very often expressed in the same way, as 9 lbs. 14 oz., are written 9¹⁴ lbs. These small figures, in the first instance, denote quarters, in the second, sixteenths.

EXAMPLES FOR DRILL.

Find the sum of —

1. $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$.
2. $\frac{3}{4}$, $\frac{1}{5}$, $\frac{5}{8}$, and $\frac{7}{9}$.
3. $\frac{5}{6}$, $1\frac{2}{3}$, $3\frac{1}{2}$, $7\frac{1}{4}$, $3\frac{2}{3}$, and $5\frac{3}{8}$.
4. $\frac{3}{5}$, $\frac{7}{9}$, $\frac{3}{4}$, $5\frac{1}{2}$.
5. $\frac{5}{14}$, $\frac{9}{15}$, $2\frac{1}{4}$, $4\frac{1}{2}$.
6. $36\frac{1}{4}$, $2\frac{5}{21}$, $6\frac{4}{27}$, $\frac{7}{45}$.

7. $41\frac{3}{4}$, $76\frac{3}{8}$, $194\frac{1}{2}$.
8. $318\frac{1}{2}$, $209\frac{1}{7}$, 315 , $\frac{17}{8}$.
9. 204 , $17\frac{9}{10}$, $\frac{5}{6}$, $3\frac{5}{8}$.
10. $415\frac{1}{2}$, $10\frac{3}{17}$, $16\frac{1}{5}$.
11. $405\frac{3}{8}$, $17\frac{1}{2}$, $901\frac{1}{8}$.
12. 3 , $\frac{7}{8}$, $\frac{9}{10}$, $\frac{1}{16}$, $\frac{1}{17}$, $\frac{4}{5}$.

13. 13^1 yds., 16^2 yds., 38 yds., 16^2 yds., 75^3 yds., 39^3 yds.
45² yds.

14. 14 yds., 38^3 yds., 29^3 yds., 16^1 yds., 22^2 yds., 38 yds., 17^2 yds., 37^3 yds.

15. 9^{15} lbs., 13^3 lbs., 16^7 lbs., 39^{13} lbs., 45^{14} lbs., 17 lbs., 48^5 lbs.

16. 38^{12} lbs., 14^6 lbs., 27^7 lbs., 38^4 lbs., 75^3 lbs., 29^4 lbs., 48^6 lbs.

17. $15\frac{2}{3}$ acres, $29\frac{3}{4}$ acres, $62\frac{5}{6}$ acres, $23\frac{5}{8}$ acres, $42\frac{9}{16}$ acres.

18. $13\frac{1}{4}$ gross, $28\frac{5}{6}$ gross, $127\frac{2}{3}$ gross, $69\frac{3}{8}$ gross, $149\frac{1}{4}$ gross.

19. $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, and $\frac{1}{8}$.

20. $\frac{5}{9}$, $17\frac{1}{2}$, $\frac{3}{8}$, $27\frac{2}{3}$, $56\frac{5}{8}$, $\frac{3}{4}$, $2\frac{7}{12}$.

SUBTRACTION OF FRACTIONS.

116. Subtraction of Fractions is the process of finding the difference between two fractions

ILLUSTRATIVE EXAMPLE.

Find the difference between $\frac{5}{6}$ and $\frac{3}{5}$.

MODEL SOLUTION.

$$\begin{array}{r} \frac{5}{6} = 25 \\ \frac{3}{5} = 18 \\ \hline \frac{7}{30} \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 30 \text{ L. C. D.} \quad \text{Ans.}$$

EXPLANATION.—The L. C. D. of the fractions is 30. Reducing the given fractions to this denominator, we have $\frac{25}{30}$ and $\frac{18}{30}$. Finding the difference of the numerators we have $\frac{7}{30}$ as the required difference of the fractions.

117. Should Mixed Numbers occur, find the difference of the fractions and the mixed numbers separately, and write the results.

ILLUSTRATIVE EXAMPLES.

Find the difference between $29\frac{5}{6}$ and $13\frac{3}{4}$.

MODEL SOLUTION.

$$\begin{array}{r} 29 \left| \frac{5}{6} = 10 \right. \\ 13 \left| \frac{3}{4} = 9 \right. \\ \hline 16 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 12 \text{ L. C. D.} \quad \frac{1}{12} + 16 = 16\frac{1}{12} \text{ Ans.}$$

Find the difference between $45\frac{7}{9}$ and $28\frac{11}{12}$.

MODEL SOLUTION.

$$\begin{array}{r} 45 \left| \frac{7}{9} + \frac{9}{9} = \frac{16}{9} = 64 \right. \\ 28 \left| \frac{11}{12} = 33 \right. \\ \hline 16 \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} 36 \text{ L. C. D.} \quad \frac{31}{36} + 16 = 16\frac{31}{36} \text{ Ans.}$$

EXPLANATION.— $\frac{7}{9}$ being less than $\frac{11}{12}$, we “borrow” 1, or $\frac{9}{9}$, from the unit order of 45, leaving that number 44. Adding $\frac{7}{9}$ and $\frac{9}{9}$, we have $\frac{16}{9}$; and subtracting $\frac{11}{12}$ from this, we have $\frac{31}{36}$. Subtracting 28 from 44 and uniting

with the difference, $\frac{31}{36}$, we have the required difference $16\frac{31}{36}$.

EXAMPLES FOR DRILL.

Find the difference between—

1. $\frac{3}{4}$ and $\frac{1}{2}$.

2. $\frac{5}{9}$ and $\frac{1}{3}$.

3. $\frac{9}{10}$ and $\frac{2}{5}$.

4. $\frac{3}{5}$ and $\frac{1}{3}$.

5. $\frac{4}{5}$ and $\frac{3}{7}$.

6. $\frac{2}{3}$ and $\frac{1}{2}$.

7. $\frac{7}{8}$ and $\frac{1}{8}$.

8. $\frac{16}{25}$ and $\frac{3}{10}$.

9. $\frac{2}{3}$ and $\frac{5}{6}$.

10. $\frac{9}{16}$ and $\frac{3}{8}$.

11. $4\frac{1}{3}$ and $2\frac{1}{4}$.

12. $4\frac{3}{8}$ and $2\frac{1}{6}$.

13. $6\frac{1}{2}$ and $1\frac{1}{3}$.

14. $1008\frac{2}{3}$ and 999.

15. 784 and $216\frac{9}{10}$.

16. $19\frac{1}{2}$ and $2\frac{3}{8}$.

17. $714\frac{1}{4}$ and $360\frac{2}{5}$.

18. $642\frac{1}{9}$ and $318\frac{3}{14}$.

19. 25^1 and 13^3 yds.

20. 38^2 yds. and 5^1 yds.

21. 194^{15} lbs. and 36^3 lbs.

TIME SAVERS.

1. To add fractions whose numerators are 1.

ILLUSTRATIVE EXAMPLE.

Find the sum of $\frac{1}{3}$ and $\frac{1}{5}$.

SOLUTION.

$$\frac{1}{3} + \frac{1}{5} = \frac{3 + 5}{15} = \frac{8}{15}$$

EXPLANATION.—Add the denominators for the new numerator, and multiply the denominators for the new denominator, which gives $\frac{8}{15}$.

2. To subtract fractions whose numerators are 1.

ILLUSTRATIVE EXAMPLE.

Subtract $\frac{1}{5}$ from $\frac{1}{3}$.

SOLUTION.

$$\frac{1}{3} - \frac{1}{5} = \frac{5 - 3}{15} = \frac{2}{15}$$

EXPLANATION.—Subtract the denominators for the new numerator, and multiply the denominators for the new denominator, which gives $\frac{2}{15}$.

EXAMPLES FOR DRILL.

1. $\frac{1}{2} + \frac{1}{3}$.

2. $\frac{1}{3} + \frac{1}{4}$.

3. $\frac{1}{3} + \frac{1}{6}$.

4. $\frac{1}{6} + \frac{1}{7}$.

5. $\frac{1}{8} + \frac{1}{19}$.

6. $\frac{1}{7} + \frac{1}{11}$.

7. $\frac{1}{2} - \frac{1}{3}$.

8. $\frac{1}{4} - \frac{1}{5}$.

9. $\frac{1}{6} - \frac{1}{7}$.

10. $\frac{1}{10} - \frac{1}{13}$.

11. $\frac{1}{11} - \frac{1}{12}$.

12. $\frac{1}{8} - \frac{1}{7}$.

MULTIPLICATION OF FRACTIONS.

118. Multiplication of Fractions is the process of finding the product of fractional numbers.

119. The process is best treated as an application of Compound Division.

$$\text{Thus, } \frac{3}{4} \times 5 = \frac{3 \times 5}{4} = \frac{15}{4} = 3 \frac{3}{4}. \quad 14 \times \frac{5}{7} = \frac{14 \times 5}{7} = 10.$$

$$\frac{3}{5} \times \frac{2}{3} \times \frac{15}{16} = \frac{\overset{3}{\cancel{3}} \times \underset{\cancel{3}}{2} \times \overset{3}{\cancel{15}}}{\underset{\cancel{5}}{5} \times \underset{\cancel{3}}{3} \times \underset{8}{\cancel{16}}} = \frac{3}{8}, \text{ etc.}$$

120. If both the factors are Mixed Numbers, reduce them to improper fractions.

$$\text{Thus, } 3\frac{1}{4} \times 2\frac{2}{3} = \frac{13}{4} \times \frac{8}{3} = \frac{26}{3} = 8\frac{2}{3}.$$

121. If one of the factors is a Mixed Number and the other a Whole Number, multiply the whole number and the fractional number separately, and add the results.

ILLUSTRATIVE EXAMPLE.

Multiply $9\frac{3}{5}$ by 7.

MODEL SOLUTION.

$$(1.) \quad 9 \times 7 = 63.$$

$$(2.) \quad \frac{3}{5} \times 7 = \frac{21}{5} = 4\frac{1}{5}.$$

$$(3.) \quad 4\frac{1}{5} + 63 = 67\frac{1}{5}. \text{ Ans.}$$

122. Compound Fractions (see Art. 92) may also be expressed as a Compound Division.

$$\text{Thus, } \frac{3}{4} \text{ of } \frac{5}{6} = \frac{3 \times 5}{4 \times 6}, \text{ or } \frac{5}{8}.$$

Hence: *To reduce compound to simple fractions, multiply together the two parts of the compound fraction.* Mathematically the word "cf" written between fractions, or between fractions and whole numbers, may be treated as expressing the multiplication of the quantities; thus, $\frac{3}{8}$ of 20 = $\frac{3}{8} \times 20$, $\frac{5}{6}$ of $2\frac{1}{2}$ = $\frac{5}{6} \times 2\frac{1}{2}$, etc.

EXAMPLES FOR DRILL.

- | | |
|--|---|
| 1. Multiply $\frac{2}{3}$ by $\frac{3}{4}$. | 8. Multiply $\frac{7}{8}$ by 9. |
| 2. Multiply $\frac{3}{4}$ by 7. | 9. Multiply $\frac{5}{8}$ by $\frac{3}{4}$. |
| 3. Multiply $\frac{5}{16}$ by $\frac{8}{9}$. | 10. Multiply $2\frac{1}{2}$ by $\frac{3}{4}$. |
| 4. Multiply $3\frac{1}{4}$ by 2. | 11. Multiply $6\frac{7}{8}$ by $\frac{3}{4}$. |
| 5. Multiply $1\frac{5}{6}$ by $\frac{8}{9}$. | 12. Multiply $3\frac{1}{4}$ by $2\frac{1}{8}$. |
| 6. Multiply $1\frac{1}{2}$ by $3\frac{1}{4}$. | 13. Multiply $3\frac{1}{6}$ by $3\frac{1}{4}$. |
| 7. Multiply $2\frac{3}{5}$ by $4\frac{3}{8}$. | 14. Multiply $2\frac{3}{4}$ by $7\frac{3}{8}$. |

Find the value of —

- | | |
|--|--|
| 15. $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{1}{2}$. | 18. $\frac{1}{4}$ of $4\frac{1}{2}$ of $\frac{3}{7}$ of $6\frac{1}{4}$. |
| 16. $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{1}{4}$ of $\frac{3}{5}$. | 19. $\frac{3}{8}$ of $\frac{2}{3}$ of $\frac{5}{6}$ of $\frac{1}{2}$ of $\frac{3}{8}$ of $\frac{1}{2}$. |
| 17. $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{5}{6}$ of $\frac{1}{2}$. | 20. $\frac{2}{3}$ of $4\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{7}{8}$ of $\frac{3}{16}$. |

MIXED NUMBERS.

- | | |
|-------------------------------------|---|
| 1. Multiply $241\frac{1}{2}$ by 8. | 6. Multiply $19\frac{1}{2}$ by $24\frac{3}{4}$. |
| 2. Multiply 78 by $4\frac{2}{3}$. | 7. Multiply $18\frac{1}{4}$ by $15\frac{1}{3}$. |
| 3. Multiply $316\frac{2}{3}$ by 9. | 8. Multiply $36\frac{7}{8}$ by $14\frac{2}{3}$. |
| 4. Multiply 97 by $6\frac{3}{8}$. | 9. Multiply $13\frac{1}{2}$ by $16\frac{3}{4}$. |
| 5. Multiply $415\frac{1}{8}$ by 15. | 10. Multiply $98\frac{1}{2}$ by $17\frac{1}{4}$. |

TIME SAVERS.

123. In multiplying mixed numbers together, it is usually better to reduce them to improper fractions. In certain cases, however, this is unnecessary.

1. To multiply mixed numbers when the whole numbers are alike, and the sum of the fractions is 1

ILLUSTRATIVE EXAMPLE.

Multiply $8\frac{1}{4}$ by $8\frac{3}{4}$.

$\frac{1}{4} \times \frac{3}{4} = \frac{3}{16}$, and $9 \times 8 = 72$. $72 + \frac{3}{16} = 72\frac{3}{16}$. EXPLANATION.—Multiply the fractions which gives $\frac{3}{16}$, add 1 to the multiplier, 8, which gives 9, then multiply $8 \times 9 = 72$, prefix it to the fraction, which gives $72\frac{3}{16}$.

2. To multiply mixed numbers, each containing $\frac{1}{2}$

ILLUSTRATIVE EXAMPLE.

Multiply $7\frac{1}{2}$ by $4\frac{1}{2}$.

SOLUTION.

$$7\frac{1}{2} \times 4\frac{1}{2} = (\frac{1}{2} \times \frac{1}{2}) + (4 \times 7) + \frac{1}{2}(4 + 7) = \frac{1}{4} + 28 + 5\frac{1}{2} = 33\frac{3}{4}.$$

EXPLANATION.—Multiply the fractions $\frac{1}{2} \times \frac{1}{2}$ which gives $\frac{1}{4}$; then to the product of 7×4 add $\frac{1}{2}$ of the sum of the whole numbers ($5\frac{1}{2}$) = $33\frac{3}{4}$.

3. To multiply mixed numbers where both fractions are alike.

ILLUSTRATIVE EXAMPLE.

Multiply $8\frac{2}{3}$ by $9\frac{2}{3}$.

SOLUTION.

$$8\frac{2}{3} \times 9\frac{2}{3} = (\frac{2}{3} \times \frac{2}{3}) + (8 + 9) \times \frac{2}{3} + (8 \times 9) = \frac{4}{9} + 11\frac{1}{3} + 72 = 83\frac{7}{9}.$$

EXPLANATION.—Multiply the fractions ($\frac{2}{3} \times \frac{2}{3}$) = $\frac{4}{9}$, then ($\frac{2}{3} \times 8$) + ($\frac{2}{3} \times 9$) = $17 \times \frac{2}{3} = 11\frac{1}{3}$, to these products add the product of the whole numbers

EXAMPLES FOR DRILL.

1. $6\frac{3}{4} \times 5\frac{3}{4}$.

2. $9\frac{2}{3} \times 9\frac{2}{3}$.

3. $3\frac{6}{7} \times 3\frac{6}{7}$.

4. $5\frac{1}{2} \times 6\frac{1}{2}$.

5. $8\frac{1}{2} \times 9\frac{1}{2}$.

6. $12\frac{1}{2} \times 7\frac{1}{2}$.

7. $9\frac{1}{2} \times 11\frac{1}{2}$.

8. $7\frac{1}{4} \times 8\frac{1}{4}$.

9. $3\frac{1}{3} \times 9\frac{1}{3}$.

10. $17\frac{3}{4} \times 8\frac{3}{4}$.

11. $7\frac{1}{2} \times 18\frac{1}{2}$.

12. $14\frac{2}{3} \times 14\frac{2}{3}$.

13. $8\frac{2}{5} \times 8\frac{2}{5}$.

14. $13\frac{1}{4} \times 7\frac{1}{4}$.

15. $19\frac{1}{8} \times 16\frac{1}{8}$.

DIVISION OF FRACTIONS.

124. Division of Fractions is the process of finding the quotient when either dividend or divisor, or both of them, are fractional quantities.

125. The Reciprocal of a quantity is the quotient arising from dividing 1 by that quantity

Thus, the reciprocal of 6 is $1 \div 6 = \frac{1}{6}$. The reciprocal of 9 is $\frac{1}{9}$, etc.

126. The Reciprocal of a Fraction is expressed by *reversing the terms of the fraction* or “inverting” the fraction. Thus, $1 \div \frac{2}{3} = \frac{3}{2} \div \frac{2}{3} = 3 \div 2 = \frac{3}{2}$; hence, $1 \div \frac{2}{3} = \frac{3}{2}$

PRINCIPLE.

Multiplying by the reciprocal of a quantity is equivalent to dividing by the quantity. Thus, $1 \div \frac{2}{3} = \frac{3}{2}$, therefore $5 \div \frac{2}{3} = 5 \times \frac{3}{2}$. Hence the —

GENERAL RULE FOR THE DIVISION OF FRACTIONAL QUANTITIES.—
Multiply the dividend by the reciprocal of the divisor.

ILLUSTRATIVE EXAMPLES.

Divide 15 by $\frac{2}{3}$.

MODEL SOLUTION.

$$1 \div \frac{2}{3} = \frac{3}{2}. \quad 15 \times \frac{3}{2} = \frac{45}{2} = 22\frac{1}{2}. \text{ Ans.}$$

Divide $8\frac{1}{3}$ by 6.

MODEL SOLUTION.

$$1 \div 6 = \frac{1}{6}. \quad 8\frac{1}{3} \times \frac{1}{6} = \frac{25}{18} = 1\frac{7}{18}. \text{ Ans.}$$

Divide $10\frac{2}{3}$ by $12\frac{4}{5}$.

MODEL SOLUTION.

$$10\frac{2}{3} \div \frac{64}{5} = \frac{64}{5}. \quad 1 \div \frac{64}{5} = \frac{5}{64}. \quad 10\frac{2}{3} \times \frac{5}{64} = \frac{5}{6}. \text{ Ans.}$$

NOTE.—When dividing a fraction by a whole number, we may employ Gen. Prin. b, Art. 102, and either divide the numerator or multiply the denominator by the whole number.

EXAMPLES FOR DRILL.

- | | | |
|---------------------------------------|---|---|
| 1. $\frac{2}{3}$ by $\frac{3}{4}$. | 9. $3\frac{1}{8}$ by 9. | 17. $19648\frac{1}{2}$ by 17. |
| 2. $\frac{1}{2}$ by $\frac{3}{8}$. | 10. 516 by $\frac{3}{4}$. | 18. 1832 by $9\frac{1}{8}$. |
| 3. $\frac{5}{7}$ by $\frac{5}{8}$. | 11. $1\frac{2}{3}$ by 4. | 19. $75\frac{1}{2}$ by $14\frac{1}{4}$. |
| 4. 5 by $1\frac{5}{6}$. | 12. $\frac{1}{2}$ by $\frac{7}{8}$. | 20. $344\frac{1}{2}$ by $8\frac{1}{3}$. |
| 5. $\frac{3}{8}$ by 4. | 13. 448 by $2\frac{1}{2}$. | 21. $984\frac{2}{3}$ by $22\frac{1}{6}$. |
| 6. $\frac{7}{8}$ by $1\frac{1}{4}$. | 14. $364\frac{1}{2}$ by 4. | 22. $9321\frac{5}{6}$ by $324\frac{1}{3}$. |
| 7. 14 by $2\frac{1}{2}$. | 15. $965\frac{2}{3}$ by 17. | 23. $3254\frac{5}{12}$ by $66\frac{2}{3}$. |
| 8. $1\frac{1}{3}$ by $3\frac{1}{3}$. | 16. $1924\frac{2}{3}$ by $7\frac{1}{8}$. | 24. $9324\frac{3}{4}$ by $64\frac{1}{2}$. |

REVIEW EXAMPLES IN COMMON FRACTIONS.

1. Find the sum and difference of $3\frac{1}{3}$ and $2\frac{1}{4}$.
2. I bought a farm for \$2,500, and sold it for $\frac{9}{10}$ of what it cost. What did I receive for the farm?
3. A man had \$15,000, and spent $\frac{2}{5}$ of what he had in building a house. What was the cost of the house?
4. Find the cost of $26\frac{1}{2}$ acres of land at \$25.50 an acre?
5. If a bushel of wheat is worth 50 cts., what is $\frac{2}{3}$ of a bushel worth?
6. If a man can walk $25\frac{1}{2}$ miles in a day, how far can he walk in $6\frac{1}{2}$ days at the same rate?
7. If $3\frac{1}{2}$ bu. of potatoes cost \$2.17, find the cost of $12\frac{3}{4}$ bu. at the same rate.
8. If 14 bu. of apples can be bought for \$3.50, how many bushels can be bought for \$ $\frac{3}{4}$?
9. At the rate of $4\frac{1}{2}$ miles per hour, how long will it take a person to walk $33\frac{1}{4}$ miles?
10. A man invested $\frac{2}{3}$ of his money in a farm, $\frac{1}{2}$ of the remainder in farm stock, and the remainder, which was \$956, in implements. What was the amount of each investment, and the total amount?
11. I bought $4\frac{1}{2}$ acres of land, and divided it into building lots of $\frac{3}{16}$ acres each. How many building lots did I obtain?
12. A man bought $133\frac{1}{2}$ yds. of cloth at one time, at another $113\frac{3}{4}$ yds., at another $317\frac{1}{4}$ yds., and sold at one time $34\frac{3}{4}$ yds., at another $265\frac{1}{4}$ yds. How many yards had he remaining?
13. If $\frac{7}{8}$ of a bushel of potatoes, worth 64 cts. a bushel, is exchanged for $6\frac{1}{8}$ lbs. of pork, what price is charged per pound for the pork?
14. A farmer gave $25\frac{1}{2}$ bu. of wheat at 48 cts. a bushel, $42\frac{3}{4}$ bu. of oats at $33\frac{1}{3}$ cts., and $36\frac{1}{4}$ bu. of corn at $62\frac{1}{2}$ cts., for $25\frac{1}{2}$ yds. of calico, at 10 cts., 40 yds. of sheeting at $24\frac{3}{4}$ cts., $45\frac{1}{2}$ yds. of alpaca at 25 cts. What amount was still due the farmer?
15. A capitalist invested $\frac{1}{8}$ of his money in farm property, $\frac{1}{2}$ of the remainder in city property, $\frac{1}{2}$ of what still remained in stocks and bonds,

and then had \$14,000 left. What was the amount of each investment, and what amount had he at first?

16. If I sold $\frac{2}{3}$ of a farm of $136\frac{1}{2}$ acres, how many acres had I left?

17. A man bought $24\frac{1}{2}$ yds. of sheeting, at $12\frac{3}{4}$ cts. per yard. If he had paid $1\frac{1}{2}$ cts. more per yard, how many yards would he have received?

18. A man bought 8 lbs. of coffee at $37\frac{1}{2}$ cts., $8\frac{3}{4}$ bu. of potatoes at 60 cts., and gave in payment a \$10 bill. How much change should he receive?

19. A farmer sowed $\frac{2}{3}$ of his farm to wheat, $\frac{1}{2}$ of the remainder to corn, $\frac{1}{2}$ of the remainder to oats, and has $14\frac{1}{2}$ acres left. How many acres are there in the farm?

20. A can do a job of work in 6 days, B in 8 days; in what time would the work be done, both working together?

SUGGESTION:—Find what part of the work both men will do in 1 day.

21. A tank has two inlets by means of which it can be filled, by one of them in 10 minutes and by the other in 12 minutes. If both inlets be open at once, how long would it take to fill the tank?

22. I bought 7 bags of oats, weighing respectively, including the bags, $86\frac{1}{4}$ lbs., $97\frac{3}{4}$ lbs., $89\frac{3}{8}$ lbs., $79\frac{7}{8}$ lbs., $84\frac{1}{4}$ lbs., $82\frac{7}{8}$ lbs., and $92\frac{3}{4}$ lbs., at 32 cts. a bushel of 32 lbs. Allowing $1\frac{1}{4}$ lbs. for each bag, what was the total cost?

23. A man and a boy working together can complete a job in 15 days; the man working alone requires 20 days to complete the job. How long will it take the boy working alone?

24. A can do a piece of work in 6 days, B in 8, and C in 10 days; if all work together, how long would it take them to complete the work?

25. A man owned $\frac{3}{4}$ of a factory and sold $\frac{7}{8}$ of his share for \$42,420. What part of the factory does he still own, and at the above rate, what is it worth?

26. A grocer bought 3 bags of coffee, the net weight of each being $134\frac{1}{2}$ lbs., $137\frac{3}{4}$ lbs., $133\frac{3}{4}$ lbs., paying $22\frac{1}{2}$ cts. a pound, and sold it for 35 cts. a pound. What was his gain?

27. At $\frac{3}{8}$ of a cent a pound, how many pounds of cement can be bought for \$21.15?

28. Bought potatoes at $\frac{3}{4}$ of a cent a pound and sold at $\frac{5}{6}$ of a cent; how many pounds must I buy and sell to gain \$100?

29. Bought 28600 lbs. of wool at $16\frac{3}{8}$ cts., and sold at $14\frac{3}{4}$ cts.; what sum did I lose?

30. There are 5 poles, whose heights respectively are as follows: $18\frac{2}{3}$ ft., $22\frac{3}{4}$ ft., $15\frac{5}{9}$ ft., $25\frac{5}{6}$ ft., and $23\frac{1}{2}$ ft. What is the average height of the poles?

31. A's weekly wages are \$13.50. What is due him at the end of $19\frac{3}{4}$ working-days?

32. $57\frac{3}{4}$ lbs. of pork at $8\frac{1}{2}$ cts. was exchanged for butter at 17 cts. per pound. Required the weight of the butter.

33. Bought 1680 bu. of corn at $3\frac{1}{2}$ cts. less than the market-price; paid charges amounting to \$19.65, and then sold the corn at $2\frac{5}{8}$ cts. more than the market-price. Find my gain.

RELATION OF NUMBERS.

127. By the **Relation of Numbers** is meant the comparison of one number with another. Thus, the relation of 6 to 12, is $\frac{1}{2}$; of 9 to 15, is $\frac{3}{5}$; of 30 to 5, is 6, etc.

128. The **Base** is the number with which another number is compared. Thus, in the expression, "8 acres is $\frac{2}{3}$ of 12 acres," 12 acres is the *base*.

129. The **Part** is the number that is compared with the base. Thus, in the expression, "\$9 is $\frac{3}{5}$ of \$15," \$9 is the *part*.

130. The **Relation** is the whole number or fraction that expresses the ratio, or relation, of the part to the base. Thus, in the expression, "\$12 is $\frac{2}{3}$ of \$18," $\frac{2}{3}$ is the *relation*.

131. In comparing two quantities, *either* may be regarded as the base, but a *change in the order of the comparison reverses the relation*.

For example, the expression, "\$9 is $\frac{3}{4}$ of \$12," may be reversed thus, "\$12 is $\frac{4}{3}$ of \$9;" "John's money is $\frac{2}{3}$ of Henry's," "Henry's money is $\frac{3}{2}$ of John's;" "6 lbs. is $\frac{1}{2}$ of 12 lbs.;" "12 lbs. is 2 ($\frac{2}{1}$) times 6 lbs."

PRINCIPLES.

1. *The Base multiplied by the Relation equals the Part.*
2. *The Part divided by the Relation equals the Base.*
3. *The Part divided by the Base equals the Relation.*

132. Problems involving the Relation of Numbers may be solved either by Logical Analysis, or by Formulas drawn from the above Principles. Students are advised to solve by both methods, full illustrations of which are given.

133. To Find the Part when the Relation and Base are given.

ILLUSTRATIVE EXAMPLE.

What is $\frac{5}{9}$ of \$72?

LOGICAL ANALYSIS.—One ninth of \$72 is \$8; and five ninths is 5 times \$8, or \$40.

FORMULA.—Base \times Relation = Part (Prin. 1.).

$$\$72 \times \frac{5}{9} = \$40.$$

EXAMPLES FOR DRILL.

- | | |
|---|--|
| 1. Find $\frac{12}{35}$ of \$701.05. | 4. Find $\frac{15}{19}$ of \$5776. |
| 2. What is $\frac{3}{4}$ of $2\frac{1}{2}$ ft.? | 5. $\frac{18}{23}$ of \$92.46 equals what sum? |
| 3. Find $\frac{13}{23}$ of \$123.75. | 6. Find $\frac{5}{11}$ of \$7 $\frac{1}{3}$. |

134. To find the Base when the Part and the Relation are given.

ILLUSTRATIVE EXAMPLE.

\$18 is $\frac{3}{4}$ of what sum?

LOGICAL ANALYSIS.—\$18 \div 3 = \$6 = $\frac{1}{4}$ of the required sum, and $\frac{4}{4}$, or the sum, is \$6 \times 4, or \$24.

ANOTHER ANALYSIS (see Art. 131).—If \$18 is $\frac{3}{4}$ of a certain sum, that sum is $\frac{4}{3}$ of \$18 = \$24, the required sum.

FORMULA.—Part \div Relation = Base (Prin. 2).

$$\text{\$18} \div \frac{3}{4} = \text{\$24. Ans.}$$

EXAMPLES FOR DRILL.

- | | |
|--|---|
| 1. \$204.85 is $\frac{17}{19}$ of what sum? | 4. \$91.65 is $\frac{13}{20}$ of what sum? |
| 2. $2\frac{3}{4}$ is $\frac{1}{3}$ of what number? | 5. $17\frac{1}{2}$ is $2\frac{1}{2}$ times what number? |
| 3. $4\frac{1}{2}$ is $\frac{3}{7}$ of what number? | 6. $8\frac{3}{4} = \frac{5}{11}$ of what number? |

135. To find the Relation when the Part and Base are given.

ILLUSTRATIVE EXAMPLES.

\$5 is what part of \$8?

LOGICAL ANALYSIS.—\$1 is $\frac{1}{8}$ of \$8. Then \$5 is $5 \times \frac{1}{8}$, or $\frac{5}{8}$, of \$8.

FORMULA.—Part \div Base = Relation.

$$\text{\$5} \div \text{\$8} = \frac{5}{8} \text{ Ans.}$$

$\frac{3}{4}$ bu. is what part of 7 bu.?

LOGICAL ANALYSIS.—1 bu. is $\frac{1}{7}$ of 7 bu. Then $\frac{3}{4}$ bu. is $\frac{3}{4} \times \frac{1}{7}$, or $\frac{3}{28}$, of 7 bu.

FORMULA.—Part \div Base = Relation.

$$\frac{3}{4} \text{ bu.} \div 7 \text{ bu.} = \frac{3}{28} \text{ Ans.}$$

$\frac{2}{3}$ lb. is what part of $\frac{3}{4}$ lb.?

LOGICAL ANALYSIS.—1 lb. is $\frac{4}{3}$ of $\frac{3}{4}$ lb. (Art. 131). Then $\frac{2}{3}$ lb. is $\frac{2}{3} \times \frac{4}{3}$, or $\frac{8}{9}$, of $\frac{3}{4}$ lb.

FORMULA.—Part \div Base = Relation (Prin. 3).

$$\frac{2}{3} \text{ lb.} \div \frac{3}{4} \text{ lb.} = \frac{8}{9} \text{ lb. Ans.}$$

NOTE.—In practice, the problems of this case are best solved by means of the Formulas, yet the logical analysis affords fine drill, and should be fully mastered.

EXAMPLES FOR DRILL.

1. $\frac{3}{4}$ of an acre is what part of 9 acres?
2. $7\frac{1}{3}$ in. is what part of 12 in.?
3. \$5 is what part of $7\frac{1}{3}$?
4. $4\frac{1}{3}$ oz. is what part of $9\frac{5}{6}$ oz.?
5. $\frac{3}{4}$ lb. is what part of $2\frac{1}{2}$ lbs.?

GENERAL PROBLEMS.

1. If $\frac{7}{8}$ of a ship is worth \$3584 more than $\frac{7}{12}$ of the same ship, what is the value of $\frac{3}{4}$ of the ship?

2. A's commissions were $\frac{2}{25}$ of his collections. If he turned over to his principal \$4692.69, after reserving his commissions, what sum did he collect, and what was the amount of his commissions?

3. Bought a $\frac{2}{3}$ interest in a threshing-machine, and sold $\frac{1}{4}$ of my share for \$75.50. What was the value of the machine?

4. What sum is $\frac{1}{5}$ less than \$73.20?

SUGGESTION.— $\frac{1}{5}$ less than a number, implies $\frac{4}{5}$ of that number; also $\frac{1}{5}$ more than a number, implies nine eighths of that number.

5. If 7 chestnuts are worth $10\frac{2}{7}$ walnuts, how many chestnuts would pay for 9 walnuts?

6. What number is $\frac{1}{6}$ more than 564?

7. \$28.50 is $\frac{1}{3}$ less than what sum?

8. A man sold $48\frac{1}{3}$ acres from a tract of land containing $265\frac{5}{6}$ acres. What part of the tract remained unsold?

9. John's money is $\frac{5}{9}$ of James's, and both together have \$98.70. What sum has each?

10. The cost of a horse is $\frac{4}{7}$ the cost of the carriage, and the carriage costs \$42.15 more than the horse. What is the cost of each?

11. John's wages are half of his father's, and both together receive \$25.80 per week. What does each receive?

12. If a cord of wood containing 128 ft. is worth \$5.25, find the value of a pile of wood containing 96 ft.

13. If a miller takes out $\frac{1}{6}$ of every grist for toll, how many bu. of corn at 56 lbs. to the bushel, must I take to the mill in order to receive 1120 lbs. of meal?

14. A's money is $\frac{2}{3}$ of B's; B's is $\frac{3}{4}$ of C's; and C's is $\frac{4}{5}$ of D's. What is the amount of their total capital, if C has \$240.60?

15. A, B, and C are partners. A invests \$800, B \$1200, and C \$400; what part of the capital did each invest?

DECIMALS.

136. A **Decimal** is a decimal fraction whose denominator is indicated by a decimal point. Thus, $\frac{8}{10}$ is expressed decimally as, .8, $\frac{5}{100} = .05$, $\frac{256}{1000} = .256$, etc.

137. The **Decimal Places** are the figures to the right of the decimal point. Thus, in .07 there are *two decimal places*, in .515 there are *three decimal places*, etc.

138. The **Number of Places** in a decimal is equal to the number of ciphers in the denominator of its corresponding decimal fraction. Thus, *one place* indicates 10ths, *two places* 100ths, *three places* 1000ths, *four places* 10000ths, etc.

Thoroughly commit the following table.

One place	indicates	tenths.
Two places	"	hundredths.
Three	"	thousandths.
Four	"	ten-thousandths.
Five	"	hundred-thousandths.
Six	"	millionths.
Seven	"	ten-millionths.
Eight	"	hundred-millionths.
Nine	"	billionths.

139. If the numerator of a decimal fraction contains *fewer figures than there are ciphers in its denominator*, it becomes necessary to *prefix*

ciphers to the numerator, in order to express its decimal. Thus, in the decimal fraction $\frac{58}{100000}$, it is necessary to prefix three ciphers to the numerator in order to express the decimal .00058.

NOTE.—It will be perceived by the observant student that the expression of decimals is merely an application of the methods of simple division, where the divisor is some power of 10; as, 100, 1000, 10000, etc.

140. Improper Decimal Fractions contain more figures in their numerators than there are ciphers in their denominators; as, $\frac{2537}{1000}$, $\frac{225}{16}$, $\frac{134}{100}$, etc.

141. Such fractions expressed decimally become **Mixed Decimals**, consisting of a whole number and a decimal; thus, $\frac{2467}{100} = 24.67 = 24$ units and 67 hundredths.

NUMERATION OF DECIMALS.

To read any ordinary decimal at sight, the student should be able to state instantly the *number of ciphers* in any given power of ten, say to billions. Study the table in Article 139 until you can do this.

ILLUSTRATIVE EXAMPLES.

Read .01026.

EXPLANATION.—Read the numerator, 1026, as you would read any other whole number; then, since there are five decimal places, the denominator is “hundred thousandths,” and is so read.

Read 2478.0000516.

EXPLANATION.—First read the whole number, 2478, then the numerator of the decimal, 516. There being seven places, the denominator of the decimal is “ten millionths,” and is so read.

EXAMPLES FOR DRILL.

Read the following decimals:—

1. .4.	7. .0153267.	13. 300.003.
2. .648.	8. 26.000124.	14. 4300.4300.
3. .038.	9. 560.1201678.	15. .00003.
4. .0012.	10. 2478.0001345.	16. .300.
5. .01201.	11. 10102.01200120.	17. .000400.
6. .0120.	12. 876.000001456.	18. .1004.

NOTATION OF DECIMALS.

ILLUSTRATIVE EXAMPLES.

142. To write decimals readily, it is necessary to have the table in Art. 139 thoroughly committed.

Write 2561 millionths.

EXPLANATION.—Write the decimal point first; then, observing that there are six decimal places (millionths) required, and that there are two more places than there are figures in the numerator of the decimal, write two ciphers and follow these with the given numerator. Do *not* write the decimal numerator first, and afterward the prefixed ciphers, for that is an old but awkward method, and altogether unnecessary.

Write 3567 and 54 ten millionths.

EXPLANATION.—Here, write the whole number, 3567, first; then the decimal point (.); then the five prefixed ciphers, 00000; and lastly, the numerator, 54. The result is 3567.0000054.

EXAMPLES FOR DRILL.

Write decimally the following:—

- 1 5 tenths.
2. 17 hundredths.
3. 7 hundredths.
4. 124 thousandths.
5. 3 thousandths.
6. 304 thousandths.
7. 564 ten-thousandths.
8. 456 ten-thousandths.
9. 498 hundred-thousandths.
10. 3648 hundred-thousandths.
11. 395 millionths.
12. Sixteen, and one hundred forty-two thousandths.
13. Twenty, and eight-hundredths.
14. Forty-two, and two hundred four ten-thousandths.
15. One hundred, and thirty-six thousand four hundred eighteen millionths.
16. Sixteen, and one hundred sixty-four thousandths.

REDUCTION OF DECIMALS.

143. The principles governing the reduction of common fractions, apply to the reduction of decimals.

TO REDUCE COMMON FRACTIONS TO DECIMALS.

144. This case is an application of Art. 108.

ILLUSTRATIVE EXAMPLE.

Reduce $\frac{3}{8}$ to a decimal.

OPERATION. $1000 \div 8 = 125.$ $\frac{3 \times 125}{8 \times 125} = \frac{375}{1000} = .375.$ EXPLANATION.—As 1000 is the lowest power of 10 that will exactly contain 8, the fraction $\frac{3}{8}$ is reduced to 1000ths, as in Art. 108. In practice, this operation is shortened by annexing ciphers to the numerator, and dividing by the denominator. Thus, $\frac{3.000}{8} = .375.$

RULE.—Annex as many ciphers to the numerator of the given fraction, as there are places in the required decimal, then divide by the denominator of the given fraction.

EXAMPLES FOR DRILL.

Reduce the following to equivalent decimals:—

1. $\frac{1}{8}$ | 2. $\frac{4}{5}$ | 3. $\frac{3}{4}$ | 4. $\frac{4}{25}$ | 5. $\frac{13}{256}$ | 6. $\frac{15}{32}$ | 7. $\frac{7}{8}$ | 8. $\frac{3}{25}$ | 9. $\frac{3}{16}$ | 10. $\frac{1}{20}$

NOTE.—To convert a mixed number into a mixed decimal, reduce the fraction to a decimal, then prefix the integer.

Reduce the following to mixed decimals:—

1. $11\frac{1}{4}$ | 2. $17\frac{1}{8}$ | 3. $14\frac{3}{16}$ | 4. $15\frac{1}{2}$ | 5. $16\frac{3}{5}$ | 6. $15.02\frac{1}{2}$ | 7. $14.03\frac{1}{8}$

TO REDUCE DECIMALS TO COMMON FRACTIONS.

ILLUSTRATIVE EXAMPLE.

Reduce .375 to a common fraction in its lowest terms.

OPERATION. $.375 = \frac{375 \div 125}{1000 \div 125} = \frac{3}{8}.$ Ans. EXPLANATION.—The decimal is reduced to a decimal fraction by supplying its denominator, 1000. The

decimal fraction, $\frac{375}{1000}$, is then reduced to its lowest terms by dividing by the G. C. D. of its terms, as in Art. 106.

RULE.—*Supply the denominator of the decimal, and reduce the resulting fraction to its lowest terms.*

ANNEXING CIPHERS.

145. The value of a decimal is not affected by annexing ciphers
Thus, $.50 = \frac{50}{100} = \frac{5}{10} = .5$.

146. Annexing ciphers to a decimal is equivalent to multiplying both terms of the decimal fraction by 10, 100, etc., and therefore (Art. 102, Gen. Prin. c.) does not change its value. Conversely, dropping ciphers from the right of a decimal is equivalent to reducing the decimal fraction to lower terms, and does not alter its value.

Reduce the following decimals to common fractions:—

1. .625.	5. 1.31625.	9. .06.	13. 16.255.
2. .075.	6. .925.	10. 14.375.	14. .0256.
3. 5.0125.	7. 24.55.	11. .008.	15. .941964.
4. 15.0375.	8. .675.	12. 14.34375.	16. .1087.

COMPLEX DECIMALS.

147. A Simple Decimal is one whose numerator is a whole number; as, .054, .7, .002.

148. A Complex Decimal is one whose numerator is a fractional number. Thus, $.02\frac{1}{3}$, $.005\frac{3}{4}$, $.0\frac{1}{6}$, are *complex decimals*.

149. A Fraction or a Complex Decimal is Reducible to a simple decimal, only when the denominator of the fraction contains no other prime factors than 2 or 5. Thus, $\frac{1}{2}$, $\frac{5}{16}$, $\frac{7}{20}$, $.013\frac{1}{8}$, $.5\frac{11}{40}$, $.0\frac{4}{25}$ are all reducible to simple decimals, since the denominators, 2, 16, 8, 20, 40, and 25, containing no other prime factors than 2 or 5, may be exactly contained in some power of 10.

150. A fraction or complex decimal is **not reducible** to a simple decimal, when the denominator of the fraction contains other factors

than 2 or 5. Thus, $\frac{2}{3}$, $\frac{5}{7}$, $\frac{7}{15}$, $.03\frac{7}{22}$, $.52\frac{7}{12}$, are not reducible to simple decimals.

NOTE.—The statements in Articles 148 and 149 apply only to fractions in their lowest terms.

EXERCISE.

Determine by inspection which of the following fractions are, and which are not, reducible to simple decimals.

$$\frac{11}{12}, \frac{15}{22}, \frac{9}{40}, \frac{19}{80}, \frac{17}{25}, \frac{11}{28}, \frac{7}{24}, \frac{11}{32}, \frac{59}{400}, \frac{87}{75}, \frac{18}{25}, \frac{5}{18}, \frac{49}{60}, \frac{19}{20}, \frac{3}{16}.$$

151. In reducing to a decimal any fraction whose denominator contains other prime factors than 2 or 5, certain figures in the decimal will be repeated if the division be carried far enough. Thus, $\frac{2}{3} = .66666$ etc., $\frac{5}{7} = 714285\bar{7}14285$ etc., $\frac{5}{12} = .416666$, etc.

Such decimals are called **Circulates** or **Circulating Decimals**; and the figure or figures repeated, the **Repetend**.

152. The incomplete division in a circulate may be indicated by the sign +, or by writing the remaining fraction. Thus, $\frac{1}{3} = .33\frac{1}{3}$, or $.33 +$; $\frac{3}{7} = .42\frac{6}{7}$, or $.42 +$, etc.

EXAMPLES FOR DRILL.

1. Change the following fractions to decimals, carrying out the division, until the repetend is reached.

(1.) $\frac{6}{7}$, (2.) $\frac{5}{9}$, (3.) $\frac{4}{11}$, (4.) $\frac{11}{12}$, (5.) $\frac{2}{7}$.

2. Reduce the following fractions to decimals of five places, ending each with the remaining fraction.

(1.) $\frac{6}{11}$, (2.) $\frac{5}{13}$, (3.) $\frac{1}{7}$, (4.) $\frac{7}{12}$, (5.) $\frac{7}{18}$.

3. Extend each of the following complex decimals to six places.

(1.) $.101\frac{3}{7}$, (2.) $.24\frac{3}{16}$, (3.) $.012\frac{1}{14}$, (4.) $.06\frac{3}{7}$, (5.) $.5\frac{5}{7}$.

4. Reduce the following complex decimals to the fractional form.

(1.) $.05\frac{3}{7}$.

SOLUTION.

$$.05\frac{3}{7} = \frac{5\frac{3}{7}}{100} = 5\frac{3}{7} \div 100 = \frac{38}{700} = \frac{19}{350}. \text{ Ans.}$$

(2.) $.81\frac{1}{8}$ | (3.) $.61\frac{1}{8}$ | (4.) $.07\frac{5}{8}$ | (5.) $.2\frac{5}{6}$ | (6.) $.08\frac{2}{3}$.

ADDITION OF DECIMALS.

153. Addition of Decimals is the process of finding the sum of two or more decimals.

ILLUSTRATIVE EXAMPLE.

Add 2.4, 16.6, .034, 16.3, $2\frac{1}{2}$, and $14.06\frac{1}{4}$.

SOLUTION.

2.4

16.6

.034

16.3

2.5

14.0625

51.8965

EXPLANATION.—Write the numbers so that like orders shall fall under like orders, and the decimal points shall be in a perpendicular column, and proceed as in simple addition, placing a point in the sum directly under the points in the addends.

RULE.—Arrange the addends so that their decimal points shall fall in a column, add as in whole numbers, and place the decimal point in the sum directly underneath the decimal point in the addends.

NOTE.—Complex decimals, if there are any, must be extended as far as the decimal places extend in any of the other numbers.

EXAMPLES FOR DRILL.

1. What is the sum of 4.38, 6.29, 25.34, 17.0128, and 14.312?
2. What is the sum of 634.1, 5.0029, 364.6, 199.4, and 348.0064 ?
3. What is the sum of 9001.48, $.006\frac{1}{4}$, 347.483, 954.2691, and 348.26 ?
4. What is the sum of $32.81\frac{3}{4}$, $45.648\frac{3}{8}$, 194, $14.36\frac{1}{8}$, $25.3486\frac{1}{2}$, $23.4\frac{3}{5}$, and $29.2\frac{1}{2}$?
5. A man cut 3.625 cords of wood on Monday, 2.075 cords on Tuesday, 1.875 cords on Wednesday, 4.0375 cords on Thursday, 5.1 cords on Friday, and 3.415 cords on Saturday. How many cords of wood did he cut during the week?
6. A farmer had 316.375 acres of land, and bought at different times 404.35 acres, 116.005 acres, 756.2 acres, and 340.8 acres. How many acres had he in all?

7. A man traveled 28.25 miles on the first day, $34\frac{1}{5}$ miles on the second day, $306.5\frac{1}{4}$ miles on the third day, 405.625 miles on the fourth day, 301 miles on the fifth day, .875 miles on the sixth day, $2.43\frac{3}{8}$ miles on the seventh day, $364.6\frac{1}{4}$ miles on the eighth day, 40.001 miles on the ninth day, and 564.009 miles on the tenth day. How many miles did he travel in all?

8. Find the sum of $4.02\frac{1}{3}$, $15.005\frac{1}{6}$, $7.2\frac{7}{12}$.

SUGGESTION.—Extend the complex decimals until the remaining fractions are of the same order. Then proceed as in adding ordinary mixed numbers.

9. Add $.00\frac{1}{7}$, $3.1\frac{2}{3}$, $19.057\frac{1}{3}$, $6\frac{3}{7}$.

10. $\frac{2}{3} + .00\frac{5}{6} + .000\frac{1}{6} + .0000\frac{1}{3} = ?$

11. $.00\frac{1}{6} + .247\frac{1}{3} + 1.05\frac{1}{4} + 2.0\frac{1}{12} = ?$

12. $\frac{5}{6} + .78\frac{3}{8} + .134\frac{1}{3} + .72\frac{5}{8} = ?$

SUBTRACTION OF DECIMALS.

154. Subtraction of Decimals is the process of finding the difference between two decimals.

ILLUSTRATIVE EXAMPLE.

Find the difference between 14.268 and 3.4811.

SOLUTION.	EXPLANATION.—
14.268	Arrange the numbers so that like orders of the subtrahend fall directly under like orders of the minuend. Subtract as in whole numbers, placing a point in the remainder directly under the points of the subtrahend and minuend.
3.4811	
<u>10.7869</u>	

RULE.—Arrange the decimals, so that the decimal point of the subtrahend will fall directly under the decimal point of the minuend, with like orders under like orders. Subtract as in whole numbers, and place the decimal point in the remainder directly beneath those of the subtrahend and minuend.

NOTE 1.—If the minuend has not as many places as the subtrahend, annex ciphers until the deficiency is supplied.

NOTE 2.—If the minuend or subtrahend is complex, reduce to the same number of decimal places, and subtract as in mixed numbers.

EXAMPLES FOR DRILL.

Find the difference between —

- | | |
|--------------------------------|---|
| 1. 6.548 and 2.34. | 6. 25.001 and 3.968. |
| 2. 13.2 and 6.987. | 7. $15.684\frac{1}{4}$ and $12.68\frac{1}{2}$. |
| 3. 19 and $8.999\frac{1}{9}$. | 8. $18.24\frac{1}{2}$ and $16.38\frac{1}{2}$. |
| 4. 10.001 and .3987. | 9. 3 tenths and 3 thousandths. |
| 5. 28.6 and 4.567. | 10. 10, and 16 ten-thousandths. |

MULTIPLICATION OF DECIMALS.

155. Multiplication of Decimals is the process of finding the product of two or more decimals.

ILLUSTRATIVE EXAMPLE.

Multiply .36 by .6.

SOLUTION.

$\begin{array}{r} .36 \\ .6 \\ \hline .216 \end{array}$	<p>EXPLANATION.—Reducing each factor to a decimal fraction, we have $\frac{36}{100} \times \frac{6}{10} = (\text{Art. 119}) \frac{216}{1000} = .216$. It is evident that the number of ciphers in the denominator of the product will equal the sum of those in the factors.</p>
---	---

RULE.—Multiply as in multiplication of whole numbers; then, from the right of the product, point off as many figures for decimals as there are decimal places in the several factors.

NOTE.—Removing the decimal point one place to the right multiplies the decimal by 10, two places by 100, three places by 1000, and so on.

EXAMPLES FOR DRILL.

- | | |
|-------------------------------|---|
| 1. Multiply 1.648 by 11. | 8. Multiply .0024 by .0021. |
| 2. Multiply 59.248 by .25. | 9. Multiply 2.785 by 100. |
| 3. Multiply 24.0124 by .0031. | 10. Multiply 1000 by .216. |
| 4. Multiply .369 by .0045. | 11. Multiply 9.6481 by 1000. |
| 5. Multiply 6.412 by .16. | 12. Multiply $.234\frac{1}{2}$ by .25. |
| 6. Multiply 25.025 by .009. | 13. Multiply $38.4\frac{1}{4}$ by $.5\frac{3}{4}$. |
| 7. Multiply .484 by 16. | 14. Multiply $1.345\frac{1}{4}$ by $2.5\frac{1}{2}$. |

DIVISION OF DECIMALS.

156. Division of Decimals is the process of finding the quotient, when either dividend or divisor is a decimal.

There are three cases:—

157. When the decimal places (dec. pl.) in the dividend and divisor are equal.

ILLUSTRATIVE EXAMPLE.

Divide .072 by .009.

OPERATION.

$$\left[.072 \div .009 \right] = \left[\frac{72}{1000} \div \frac{9}{1000} \right] = \left[\frac{72}{1000} \times \frac{1000}{9} \right] = \frac{72}{9} = 8. \text{ Ans.}$$

EXPLANATION.—Supplying the decimal denominators, and dividing as in division of fractions, by inverting divisor, the denominators of the dividend and divisor, being alike, cancel each other, and the operation becomes $72 \div 9 = 8$. Hence, in examples in this case the quotient will have no dec. pl., and will be found by simply dividing the numerator of the dividend by the numerator of the divisor.

NOTE.—Should the division be inexact, the remaining fraction can, of course, be extended as a decimal.

158. When the dec. pl. of the dividend exceed those of the divisor.

ILLUSTRATIVE EXAMPLE.

Divide .00048 by .06.

OPERATION.

$$\left[.00048 \div .06 \right] = \left[\frac{48}{100000} \div \frac{6}{100} \right] = \left[\frac{48}{100000} \times \frac{100}{6} \right] = \frac{8}{1000} = .008. \text{ Ans.}$$

EXPLANATION.—Supplying denominators, inverting divisors, and canceling the ciphers, we have a surplus of 1000 in the dividend, which is a divisor of the integer quotient 8, and gives .008 as the result. Hence, in examples in this case the quotient will have as many dec. pl. as those in the dividend exceed those of the divisor.

159. When the dec. pl. in the divisor exceed those in the dividend.

ILLUSTRATIVE EXAMPLE.

Divide .54 by .000006.

OPERATION.

$$[.54 \div .000006] = \left[\frac{54}{100} \div \frac{6}{1000000} \right] = \left[\frac{54}{100} \times \frac{1000000}{6} \right] = 90000. \text{ Ans.}$$

EXPLANATION.—Supplying denominators, inverting divisors, and canceling, there is a surplus of 10000 in the divisor, which, through the inversion, becomes a multiplier of the integer quotient 9, giving 9×10000 , or 90000, as the result.

Hence, in examples under this case, annex as many ciphers to the integer quotient as the dec. pl. of the divisor exceed those of the dividend.

NOTE.—The ciphers may be annexed to the dividend numerator before dividing.

Thus, in the last example we might have annexed the ciphers to 54, obtaining 540000, and dividing this by 6, obtained the answer 90000. This is the better process, unless the division is exact.

160. A decimal is divided by 10, 100, 1000, etc., by moving the decimal point to the left, as many places as there are ciphers in the divisor.

161. When a decimal is to be divided by a whole number ending in ciphers, cut off the ciphers and remove the decimal point to the left, as many places as there are ciphers. Thus, to divide 2.4 by 6000, move the decimal point three places to the left, obtaining .0024; then divide this number by 6.

GENERAL RULE.

162. Divide the numerator of the dividend by the numerator of the divisor, as in whole numbers.

If there is a surplus of dec. pl. in the dividend, point off a number of places from the right of the quotient, equal to this surplus.

If there is a surplus of dec. pl. in the divisor, annex ciphers equal to this surplus, to the quotient, or to the dividend numerator before dividing.

NOTE.—If the numerator of the divisor is not contained in that of the dividend, annex ciphers to the dividend until it will contain the numerator of the divisor.

EXAMPLES FOR DRILL.

- | | |
|--------------------------|------------------------------|
| 1. Divide 8.84 by 26. | 10. Divide 532.5 by .005. |
| 2. Divide .075 by 15. | 11. Divide 3840 by 1.12. |
| 3. Divide 370.6 by .017. | 12. Divide 62.5 by 1.25. |
| 4. Divide 1.6 by .64. | 13. Divide 2.42 by .11. |
| 5. Divide .005 by 400. | 14. Divide 48.4 by 16. |
| 6. Divide 345 by .0005. | 15. Divide .0875 by .25. |
| 7. Divide 33.33 by 1.1. | 16. Divide 46.23 by 2 3. |
| 8. Divide 264 by .19. | 17. Divide 129.625 by .004. |
| 9. Divide 16.9 by .13. | 18. Divide 195.624 by .0012. |

Find the sum of the quotients.

- | | |
|-------------------|---------------------|
| 19. $2 \div .2$. | 20. $3 \div 30$. |
| $.2 \div 2$. | $.3 \div 300$. |
| $.02 \div 2$. | $3 \div 3000$. |
| $2 \div .02$. | $30 \div .3$. |
| $20 \div .2$. | $.003 \div 30$. |
| $20 \div .002$. | $.03 \div 3000$. |
| $.002 \div 20$. | $.0003 \div .01$. |
| $.02 \div .002$. | $30 \div 1000$. |
| $10 \div .2$. | $3000 \div .0003$. |
| $200 \div .2$. | $300 \div .3$. |

GENERAL PROBLEMS.

1. Reduce $\frac{19}{20}$ to a decimal fraction.
2. Reduce .625, .00375 to common fractions.
3. Multiply 1 tenth by 1 thousandth.
4. Divide 13 hundredths by 2 tenths.
5. Find the number of pounds in four coal bins, the first containing 67482.0625 lbs., the second 1340.375 lbs., the third, 119625.25 lbs., the fourth, 95678.0125 lbs.
6. Find the cost of 496.75 lbs. of sugar at $4\frac{1}{4}$ cts. per pound.
7. How many yards of cloth at 25 cts. a yard should be given in exchange for $364\frac{1}{2}$ lbs. of butter at $16\frac{3}{4}$ cts. a pound?
8. A clerk's salary is \$15.75 a week; he pays \$3.25 a week for board, gives 65 cts. a week for charity, and his other weekly expenses

are \$2.55. How much of his salary will he have left at the end of 48 weeks?

9. How many barrels of flour, worth \$5.25 a barrel, should be given for $85\frac{1}{2}$ bu. of wheat, at $48\frac{1}{2}$ cts. a bushel, and $124\frac{3}{4}$ bu. of corn, at 45 cts. a bushel?

10. Bought an invoice of tea, and sold .25 of it at one time, $.37\frac{1}{2}$ at another, and at another $.33\frac{1}{3}$ of the remainder. What part of the invoice remained?

11. An excavator received \$482.50 for excavating a cellar, at 65 cts. a cubic yard. How many cubic yards of earth were removed?

12. If 16.5 bu. of oats cost \$6.60, what would $234.7\frac{1}{2}$ bu cost?

13. A brick block in a city cost \$13,800, the lot on which the block stood cost .8 of the cost of the block, the annual rent received from the building was .0375 of the whole cost. What amount of rent did the property yield?

14. A grocer laid out in stock \$3,480; $\frac{1}{8}$ of the whole being invested in coffee at 25 cts. a pound, $\frac{1}{10}$ in tea at 40 cts. a pound, $\frac{1}{4}$ in sugar at $5\frac{1}{2}$ cts. a pound, .4 in spices at $12\frac{1}{2}$ cts. a pound, and the remainder in potatoes at 35 cts. a bushel. What quantity of each did he buy?

15. Find the cost of 17 sacks of coffee, each sack weighing $134\frac{1}{2}$ lbs., at $24\frac{1}{2}$ cts. a pound.

16. If a train travels at the rate of 25.125 miles an hour, how long will it take it to travel 1000 miles?

17. If 86.375 acres of land cost \$3,455, find the cost of $964.33\frac{1}{3}$ acres?

18. A merchant bought 6484.125 yds. of serge, at 37.5 cts. a yard, and sold it for 62.5 cts. a yard. What did he gain by the transaction?

19. If 25.2 acres of wheat produce 516.6 bu. of wheat, what would 75.4 acres produce at the same rate of production?

20. A person has an income of \$5,674, and invests .75 of it in city property, .5 of the remainder in farm property, .8 of the then remainder in a mortgage, and the remainder in grain worth 45 cts. a bushel. How much did he invest in each, and how many bushels of grain did he receive?

DENOMINATE NUMBERS.

163. Quantity is that which can be measured, as, weight, time, value, distance, or space.

164. A Standard is the unit established by law or custom for the measurement of quantity. Thus, the pound, the yard, the dollar, the day, etc., are standards.

165. Measurement Units consist of the Standard, and of units greater or less than the Standard and derived from it, for the measurement of quantities of varying amount. Thus, the mile, rod, foot, inch, etc., are measurement units derived from the standard yard; the year, month, week, hour, minute, and second, are measurement units derived from the standard day.

166. Similar Measurement Units are those units that are used in measuring quantities of the same kind. Thus the grain, the ounce, the pound, and the ton, are similar measurement units, for the measurement of weight.

NOTE.—Standards are always derived from some natural object. Thus, the yard is derived from the length of the human arm; the day from the time of the earth's daily rotation; the grain from the weight of a grain of wheat, etc.

167. A Denominate Number is any number, or combination of numbers, composed of similar measurement units. Thus, 5 ft., 7 lbs. 6 oz., 4 hours 20 min., are denominate numbers.

168. A Simple Denominate Number is one that is expressed by units of one denomination, as, 5 rds., 7 gal., 2 hrs.

169. A Compound Denominate Number is one that is expressed by units of more than one denomination; as, 2 hrs. 15 min., 5 lbs. 6 oz.

NOTE 1.—Observe that a compound denominate number expresses the measurement of a single quantity. Thus, 2 mi. 76 rds. 2 yds. 1 ft. 10 in., applies to a *single distance* and is therefore spoken of, and treated, as a *single number*

NOTE 2.—Denominate numbers are always *concrete*.

170. A Table is any list of Similar Measurement Units, together with their related values, and customary abbreviations.

171. A **Scale** is a number, or series of numbers, showing the relative value of the units used in any table. Scales are either *Uniform or Variable*.

ILLUSTRATIONS.

TABLE OF DRY MEASURE.

2 pints (pts.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

Scale: 2 - 8 - 4. (Variable.)

TABLE OF U. S. MONEY.

10 mills (m.)	= 1 cent (ct., or ¢)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar (\$)
10 dollars	= 1 eagle (E.)

Scale: 10 (uniform.)

MEASURES OF VALUE.

172. In all civilized lands, property values are measured by means of Money, and these values, as well as debts of all kinds, are expressed by means of **Monetary Units**, that vary in value and name in different countries.

173. The term **Money** implies anything authorized by law and commonly used for the purchase of property or the payment of debts.

174. **Ultimate Money** or **Money of Redemption** is always made from certain metals, chiefly gold and silver; though copper, bronze, and nickel are sometimes used for the smaller coins.

175. By the term **Legal Tender**, is meant anything that the law declares may be legally offered, or tendered, for the discharge of a debt.

NOTE.—Everything that is a legal tender, is money, but some money is not a legal tender. Thus, bank notes are money, but not a legal tender. Checks, drafts, etc., are often used for the payment of debts or the purchase of property, but, being unregulated by law, are not money.

Government notes, Gold and Silver certificates, and other forms of public indebtedness often circulate as legal tender money.

UNITED STATES MONEY.

176. United States Money is the legal tender currency of the United States.

TABLE OF U. S. MONEY.

10 mills	= 1 cent (ct., or ¢)
10 cents	= 1 dime (d.)
10 dimes	= 1 dollar (\$)
10 dollars	= 1 eagle (E.)

177. Following is a table of the classes and denominations of United States Coined Money.

COINS.

Material.	Denominations.
Gold,	\$2½, \$5, \$10 (Eagle), \$20.
Silver,	\$1, 50¢, 25¢, 10¢.
Nickel,	5¢.
Copper,	1¢.

NOTE.—The mill is merely a convenient designation for the tenth part of a cent, and is never coined.

PAPER CURRENCY.

178. The Paper Money of the United States, consists of Treasury Notes, or "Greenbacks" (legal tender), also, Gold and Silver Certificates, and National Bank Notes (payment secured by the Government).

Bills in this money are issued for various amounts, but never for a less sum than \$1.

179. The Standard Coin of the United States is at present the Gold Dollar, weighing 25.8 grains. The Silver Dollar weighs 412½ grains.

180. The weight, material, and denominations of United States coins are regulated by Congress.

Gold Coins are composed of Standard Gold, which consists of nine parts *pure gold*, and one part *alloy*; the latter is composed of copper and silver, the silver not to exceed $\frac{1}{10}$ of the alloy.

Silver Coins are composed of Standard Silver, which consists of nine parts *pure silver* and one part *pure copper*.

181. The weight of the pure silver in a silver dollar is at present about 16 times the weight of the pure gold in a gold dollar. Hence the expression "16 to 1," so much used in current coinage discussion.

182. Since the scale of the U. S. Monetary Units is a decimal one, the computations involve no principles other than those of integers and decimals.

COUNTING-HOUSE PRACTICE.

183. The purpose of this article is, 1st, to give a thorough drill in short and rapid methods of business computations; 2d, to give the student an opportunity to acquire facility in handling the aliquot parts of \$1; 3rd, to enable the student to become ready in calculations.

184. An Aliquot Part of a number is any number that will exactly divide it.

ALIUOT PARTS OF ONE DOLLAR.

50¢ = $\$ \frac{1}{2}$.	$3\frac{1}{3}$ ¢ = $\$ \frac{1}{30}$.	$8\frac{1}{3}$ ¢ = $\$ \frac{1}{12}$.
25¢ = $\$ \frac{1}{4}$.	2¢ = $\$ \frac{1}{50}$.	5¢ = $\$ \frac{1}{20}$.
20¢ = $\$ \frac{1}{5}$.	$33\frac{1}{3}$ ¢ = $\$ \frac{1}{3}$.	$2\frac{1}{2}$ ¢ = $\$ \frac{1}{40}$.
10¢ = $\$ \frac{1}{10}$.	$16\frac{2}{3}$ ¢ = $\$ \frac{1}{6}$.	$1\frac{2}{3}$ ¢ = $\$ \frac{1}{60}$.
$6\frac{1}{4}$ ¢ = $\$ \frac{1}{16}$.	$12\frac{1}{2}$ ¢ = $\$ \frac{1}{8}$.	

185. To find the cost when the price is an Aliquot Part.

ILLUSTRATIVE EXAMPLE.

Find the cost of $387\frac{1}{2}$ yds., at 50¢ a yard.

SOLUTION.
 $387\frac{1}{2}$ yds. at \$1 per yd. cost \$387.50.
 $\frac{1}{2}$ of \$387.50 = \$193.75.

EXPLANATION.—At \$1 per yard $387\frac{1}{2}$ yds. will cost \$387.50, at $\$ \frac{1}{2}$, or 50¢ per yard, $387\frac{1}{2}$ yds. will cost $\frac{1}{2}$ of \$387.50, or \$193.75.

NOTE.—In the following examples it is desired that the student do the work as far as possible by mental process. In setting down the results, five mills, or half a cent, or more, will be considered an additional cent; less than this will be omitted.

EXAMPLES FOR DRILL.

Find the cost of—

- | | |
|-----------------------|------------------------|
| 1. 316 lbs. at 50¢. | 9. 564 gal. at 25¢. |
| 2. 792 yds. at 25¢. | 10. 952 lbs. at 12½¢. |
| 3. 368 yds. at 12½¢. | 11. 520 gal. at 20¢. |
| 4. 450 lbs. at 10¢. | 12. 1248 yds. at 16⅔¢. |
| 5. 985 bbls. at 20¢. | 13. 96¾ yds. at 5¢. |
| 6. 764½ bu. at 50¢. | 14. 960 lbs. at 5¢. |
| 7. 1248 yds. at 33⅓¢. | 15. 396¼ gal. at 25¢. |
| 8. 969 yds. at 33⅓¢. | 16. 357¼ lbs. at 33⅓¢. |

186. It will be noticed that where the price is not an aliquot part of a dollar, it may be composed of two aliquot parts, as $37\frac{1}{2}\text{¢}$ = $\$ \frac{3}{8}$ = $\$ \frac{1}{4}$ + $\$ \frac{1}{8}$; so that by finding the price at $\$ \frac{1}{4}$, and then at $\$ \frac{1}{8}$, the sum of these results will be the cost at $37\frac{1}{2}\text{¢}$.

ILLUSTRATIVE EXAMPLE.

Find the cost of 320 yds. at $37\frac{1}{2}\text{¢}$ per yard.

SOLUTION.

320 yds. at $\$ \frac{1}{4}$ per yard cost \$80.320 yds. at $\$ \frac{1}{8}$ per yard cost 40.\$120.

\$40, or \$120. Or, at $\$ \frac{1}{8}$ per yard 320 yds. will cost \$40, at $\$ \frac{3}{8}$ per yard 320 yds. will cost 3 times \$40, or \$120.

EXPLANATION.—At $\$ \frac{1}{4}$ per yard320 yds. will cost \$80; at $\$ \frac{1}{8}$ per yard320 yds. will cost \$40; at $\$ \frac{3}{8}$ per yard

320 yds. will cost the sum of \$80 and

187. There are also many prices which are not aliquot parts, but which lack an aliquot part of being \$1; as $\$ \frac{5}{6}$ lacks $\frac{1}{6}$ of being \$1, or equals \$1 minus $\$ \frac{1}{6}$; then to find the price at $83\frac{1}{3}\text{¢}$, find the cost at \$1, and subtract the cost at $\$ \frac{1}{6}$.

ILLUSTRATIVE EXAMPLE.

Find the cost of 546 gal. at $83\frac{1}{3}\text{¢}$ per gallon.

SOLUTION.

546 gal. at \$1 per gallon cost \$546.

546 gal. at $\$ \frac{1}{6}$ per gallon cost 91.

\$546 — \$91 = \$455.

\$455.

EXPLANATION.—At \$1 per gal-

lon 546 gal. will cost \$546, at $\$ \frac{1}{6}$

per gallon 546 gal. will cost \$91;

then at $\$ \frac{5}{6}$ per gallon 546 gal. will

cost the difference between \$546 and \$91, or \$455. Or, at $\$ \frac{1}{6}$ per gallon 546 gal. will cost \$91, at $\$ \frac{5}{6}$ per gallon 546 gal. will cost 5 times \$91, or \$455.

EXAMPLES FOR DRILL.

Find the cost of:—

- | | |
|------------------------------------|------------------------------------|
| 1. 720 yds. at $87\frac{1}{2}$ ¢. | 7. 364 lbs. at 75 ¢. |
| 2. 1800 lbs. at $83\frac{1}{3}$ ¢. | 8. 424 gal. at $62\frac{1}{2}$ ¢. |
| 3. 906 bu. at $83\frac{1}{3}$ ¢. | 9. 374 doz. at 80 ¢. |
| 4. 1000 yds. at $36\frac{1}{4}$ ¢. | 10. 600 bu. at $83\frac{1}{3}$ ¢. |
| 5. 400 gal. at $37\frac{1}{2}$ ¢. | 11. 100 yds. at $65\frac{1}{4}$ ¢. |
| 6. 360 lbs. at $62\frac{1}{2}$ ¢. | 12. 1000 bu. at $93\frac{3}{4}$ ¢. |

188. To find the cost of articles that are sold by the ton, the 100, or by the 1000.

NOTE.—Use abbreviations thus, 100 (C), 1000 (M), ton (T).

ILLUSTRATIVE EXAMPLE.

Find the cost of 540 lbs. of flour at \$2 per C.

EXPLANATION.—540 lbs. at \$2 a pound, would cost \$1080; but since the price is \$2 a C, this will be 100 times too much; then divide \$1080 by 100, which gives \$10.80. Or, divide the number of pounds, 540, by 100, which gives \$5.40, multiplied by \$2, gives \$10.80.

RULE.—*Multiply the number of articles by the price per C, and divide by 100; or point off two places from the right of the number of articles and multiply by the price per C. When the price is given per M, multiply by the price and point off three places in the product.*

EXAMPLES FOR DRILL.

What is the cost of:—

1. 634 lbs. of flour at \$3 per C?
2. 380 lbs. of salt at \$1.50 per C?
3. 6,240 lbs. of lead at \$4.50 per C?
4. 958 fence posts at \$9.60 per C?
5. 348 fence rails at \$19 per C?
6. 9,456 lbs. of scrap iron at \$1.20 per C?
7. 6,480 ft. of lumber at \$2.50 per C?
8. 16,180 lbs. of nails at \$2.25 per C?
9. 4,824 lbs. of flour at \$2.25 per C?

10. 1,650 ft. of lumber at \$20 per M?
11. 9,480 ft. of lumber at \$25 per M?
12. 6,940 ft. of lumber at \$36 per M?
13. 4,654 ft. of lumber at \$30 per M?
14. 17,364 ft. of lumber at \$36.50 per M?
15. 22,648 ft. of lumber at \$29.50 per M?

189. To find the cost of articles when sold by the ton.

ILLUSTRATIVE EXAMPLE.

Find the cost of 2,480 lbs. of coal at \$6 per ton.

EXPLANATION.—By cutting off three places we have 2.480 lbs., and by taking one-half of the price, or \$3, this gives the price per 1000 lbs., then 2.480 multiplied by 3 equals \$7.44.

RULE.—*Cut off three places from the right of the number, and multiply by one-half of the price per ton. Or, multiply the number of pounds by the price per ton, then take one-half of the product and point off three places.*

REMARK.—Should it be more convenient to take one-half the number of pounds, and multiply this by the price per ton, the result will be the same.

EXAMPLES FOR DRILL.

Find the cost of—

1. 3,140 lbs. of soft coal at \$2 per ton.
2. 3,460 lbs. of coal at \$8 per ton.
3. 9,480 lbs. of hay at \$10 per ton.
4. 7,656 lbs. of straw at \$3.50 per ton.
5. 6,484 lbs. of coal at \$6.25 per ton.
6. 14,648 lbs. of iron at \$36.50 per ton.
7. 756 lbs. of lime at \$3.75 per ton.
8. 4,560 lbs. of coal at \$6.50 per ton.
9. 13,648 lbs. of copper at \$175 per ton.
10. 3,648 lbs. of hay at \$9.75 per ton.

190. To find the cost of given weight in pounds, when the price is given per bushel or barrel.

ILLUSTRATIVE EXAMPLE.

Find the cost of 6,480 lbs. of wheat, at 48¢ per bushel of 60 lbs.

EXPLANATION.—At 48¢ a pound, the cost of 6,480 lbs. would be \$3110.40; but the price was 48¢ per bushel of 60 lbs.; therefore the cost will be $\frac{1}{60}$ of \$3110.40, or \$51.84.

RULE.—*Multiply the number of pounds by the price per bushel or barrel, and divide the product by the number of pounds in one bushel or barrel.*

EXAMPLES FOR DRILL.

Find the cost of—

1. 728 lbs. of wheat at 45¢ per bushel of 60 lbs.
2. 3,506 lbs. of corn, in ear, at 54¢ per bushel of 70 lbs.
3. 4,850 lbs. of beans at \$1.50 per bushel of 60 lbs.
4. 7,648 lbs. of clover seed at \$6.30 per bushel of 60 lbs.
5. 3,480 lbs. of corn, in ear, at 62¢ per bushel of 70 lbs.
6. 4,672 lbs. of buckwheat at 95¢ per bushel of 56 lbs.
7. 7,640 lbs. of shelled corn at 65¢ per bushel of 56 lbs.
8. 4,285 lbs. of barley at 64¢ per bushel of 46 lbs.
9. 6,240 lbs. of oats at 35¢ per bushel of 32 lbs.
10. 3,480 lbs. of timothy-seed at \$1.75 per bushel of 45 lbs.

MENTAL EXERCISES.

191. Remember in multiplying by 10, 100, etc., to annex as many ciphers to the multiplicand as there are ciphers in the multiplier. If the multiplicand has decimal places, remove the decimal point as many places to the right as there are ciphers in the multiplier.

192. To find the cost of any number of articles at 10¢ each, cut off one place from the right of the multiplicand.

If there are decimal places in the multiplicand remove the point one place to the left.

NOTE.—In multiplying by numbers that are easily factored, use the factors; as in multiplying 17 by 24, say 17 multiplied by 4 equals 68; 68 multiplied by 6 equals 408.

193. In working the following examples, extensions should be made mentally, carrying out the exact fractions of a cent, until the final result of each problem is obtained; then, half a cent, or 5 mills, will be considered a whole cent; less than half a cent will be omitted.

EXAMPLES FOR DRILL.

Find the total cost of the following :—

1. 16 yds. at 8¢.	$3\frac{1}{2}$ yds. at 8¢.	$16\frac{1}{4}$ yds. at 8¢.
19 " " 9¢.	$7\frac{1}{2}$ " " 12¢.	$14\frac{1}{2}$ " " 12¢.
15 " " 12¢.	$15\frac{1}{2}$ " " 8¢.	$13\frac{1}{4}$ " " 9¢.
12 " " $4\frac{1}{2}$ ¢.	$19\frac{1}{2}$ " " 7¢.	$15\frac{1}{2}$ " " 11¢.
2. $17\frac{1}{2}$ yds. at 10¢.	$16\frac{1}{8}$ yds. at 12¢.	$13\frac{1}{2}$ yds. at 9¢.
$34\frac{1}{4}$ " " 10¢.	$15\frac{1}{4}$ " " 8¢.	$15\frac{1}{2}$ " " 10¢.
$25\frac{1}{2}$ " " 10¢.	$16\frac{3}{4}$ " " 12¢.	$16\frac{3}{4}$ " " 4¢.
$14\frac{1}{8}$ " " 9¢.	$19\frac{1}{2}$ " " 50¢.	$29\frac{3}{4}$ " " 10¢.
3. $3\frac{3}{4}$ yds. at 8¢.	17 yds. at 18¢.	$25\frac{1}{2}$ yds. at $12\frac{1}{2}$ ¢.
$4\frac{1}{2}$ " " 12¢.	14 " " 50¢.	96 " " $16\frac{2}{3}$ ¢.
$14\frac{3}{4}$ " " 8¢.	$56\frac{1}{2}$ " " 50¢.	366 " " $16\frac{2}{3}$ ¢.
16 " " 16¢.	$36\frac{1}{2}$ " " 50¢.	342 " " $33\frac{1}{3}$ ¢.
4. $17\frac{1}{2}$ yds. at 10¢.	$87\frac{3}{4}$ yds. at $12\frac{1}{2}$ ¢.	$457\frac{1}{2}$ yds. at $33\frac{1}{3}$ ¢.
325 " " 50¢.	$256\frac{1}{2}$ " " $16\frac{2}{3}$ ¢.	$9783\frac{3}{4}$ " " 50¢.
157 " " 25¢.	$364\frac{1}{4}$ " " 20¢.	$543\frac{1}{4}$ " " $12\frac{1}{2}$ ¢.
$97\frac{1}{2}$ " " $33\frac{1}{3}$ ¢.	$968\frac{3}{4}$ " " 25¢.	1000 " " $87\frac{1}{2}$ ¢.

NOTE.—In making the extensions in the following examples, half a cent, or 5 mills, will be considered an additional cent; less than half a cent, or 5 mills, will be omitted.

5. 384 lbs. at 10¢.	63 lbs. at 11¢.	725 lbs. at 20¢.
264 " " 5¢.	450 " " $16\frac{2}{3}$ ¢.	984 " " $33\frac{1}{3}$ ¢.
1000 " " $12\frac{1}{2}$ ¢.	964 " " 50¢.	475 " " 10¢.
1500 " " 9¢.	375 " " 50¢.	$984\frac{1}{2}$ " " $33\frac{1}{3}$ ¢.
6. 756 lbs. at $33\frac{1}{3}$ ¢.	$756\frac{1}{4}$ lbs. at 50¢.	9385 lbs. at 50¢.
2840 " " 25¢.	924 " " 25¢.	$362\frac{1}{2}$ " " 10¢.
964 " " $16\frac{2}{3}$ ¢.	368 " " 25¢.	124 " " 11¢.
756 " " $33\frac{1}{3}$ ¢.	1265 " " 25¢.	364 " " $12\frac{1}{2}$ ¢.

7. 756 lbs. at 6¢.	224½ lbs. at 10¢.	1000 lbs. at 66⅔¢.
386 " " 7¢.	964½ " " 12½¢.	2000 " " 75¢.
984½ " " 10¢.	384 " " 11¢.	4000 " " 83⅓¢.
386⅔ " " 12½¢.	678 " " 25¢.	344½ " " 33⅓¢.
8. 446½ yds. at 33⅓¢.	269¾ yds. at 12½¢.	50 yds. at \$1.25.
299½ " " 25¢.	800 " " 62½¢.	540 " " \$1.50.
368½ " " 16⅔¢.	800 " " 6¼¢.	1000 " " 64¾¢.
204½ " " 10¢.	1000 " " 37½¢.	100 " " 97¼¢.

194. To find the quantity when the price is an aliquot part of \$1.

ILLUSTRATIVE EXAMPLE.

How many gallons of syrup can be bought for \$36, if syrup is worth \$.33⅓, \$.83⅓, and \$1.50 per gallon?

SOLUTION 1.—At 33⅓¢ per gallon, \$1 will buy 3 gal., then \$36 will buy 36 times 3, or 108 gal.

SOLUTION 2.—If 1 gal. costs 83⅓¢, or \$⅘, then \$36 should buy as many gallons as 83⅓¢, or \$⅘ is contained times in \$36, or $\$36 \div \$\frac{4}{5} = \$36 \times \frac{5}{4} = 45$ gal. Or, at 83⅓¢, or \$⅘ per gallon, \$1 will buy 1⅓ gal., and \$36 will buy 36 times 1⅓ gal., or 45 gal.

SOLUTION 3.—At \$1.50 per gallon \$1 will buy ⅔ of a gallon; then \$36 will buy 36 times ⅔ gal., or 24 gal. Or, \$36 will buy as many gallons as \$1½ is contained times in \$36, or $\$36 \div \$1\frac{1}{2} = \$36 \times \frac{2}{3}$, or 24 gal.

EXAMPLES FOR DRILL.

How many pounds—

1. At 25¢ a pound can be bought for \$275?
2. At 50¢ a pound can be bought for \$248?
3. At 16⅔¢ a pound can be bought for \$924?
4. At 33⅓¢ a pound can be bought for \$756?
5. At 75¢ a pound can be bought for \$315?
6. At 50¢ a pound can be bought for \$25.40?
7. At \$1.25 a pound can be bought for \$650?
8. At \$1.50 a pound can be bought for \$64.50?
9. At 83⅓¢ a pound can be bought for \$34.80?
10. At 62½¢ a pound can be bought for \$340?
11. At 20¢ a pound can be bought for \$34.60?
12. At 12½¢ a pound can be bought for \$348.50?

BILLS AND ACCOUNTS.

195. A **Bill** is a written itemized statement of indebtedness arising from property sold or services rendered, and is given by the creditor to the debtor.

196. A **Receipted Bill** is one in which the creditor or his agent acknowledges over his signature that the indebtedness set forth in the bill has been discharged.

197. An **Invoice** is a written itemized statement of goods sold at wholesale at a particular time, and is given by the seller to the buyer.

NOTE.—The term "Bill" is often applied to an Invoice.

198. A **Statement of Account** is a bill that is rendered periodically, and lists all purchases and payments made during the period covered by the statement. Statements are usually rendered at the end of each month.

199. In making out business papers, many terms, characters, and abbreviations are used, the principal of which will be found in the following list:—

COMMERCIAL ABBREVIATIONS.

A 1.....First Quality.	Com.....Commission.	i. e.....That is.
Acct. %.....Account.	Cr.....Creditor.	In.....Inches.
Agt.....Agent.	Cs.....Cases.	I. O. U.....I owe you.
Amt.....Amount.	Cwt.....Hundredweight.	Ins.....Insurance.
Arr.....Arrive.	d.....Pence.	Inst.....Instant.
Ass'd.....Assorted.	Dft.....Draft.	Inv.....Invoice.
Bal.....Balance.	Dis.....Discount.	Inv't.....Inventory.
Bbl.....Barrel.	Do. ditto, or ("").....The same	Jr. or jun.....Junior.
Bdls.....Bundles.	Doz.....Dozen.	Lbs.....Pounds.
Bds.....Boards.	Dr.....Debtor.	Ledg. or L.....Ledger.
Bgs.....Bags.	Dr'ge.....Drayage.	L. F.....Ledger Folio.
Bkts.....Baskets.	Ds., or da.....Days.	M.....Thousand.
Blk.....Black.	Ea.....Each	Mdse.....Merchandise.
Bls.....Bales.	E. & O. E..... <div style="display: inline-block; vertical-align: middle;">Errors and Omissions Excepted.</div>	Mem.....Memorandum.
Bot.....Bought.	Exch.....Exchange.	Messrs..... <div style="display: inline-block; vertical-align: middle;">Gentlemen, or Sirs.</div>
Br.....Brig.	Fig'd.....Figured.	Mo.....Month.
Bro.....Brown or Brother.	Fir.....Firkin.	Mr.....Master, or Mister.
Brot.....Brought.	F. O. B.....Free on Board.	Mrs.....Mistress.
Bu.....Bushels.	Fol.....Folio or Page.	N. B.....Take Notice.
Bxs.....Boxes.	Fr't.....Freight.	No.....Number.
C.....Hundred.	Ft.....Foot or Feet.	Oz.....Ounces.
¢ or cts.....Cents.	Gal.....Gallon.	P.....Page.
Chgd.....Charged.	Gr. or gro.....Gross.	Pp.....Pages.
Chts.....Chests.	Guar.....Guarantee.	Pay't or Paym't..... <div style="display: inline-block; vertical-align: middle;">Pay- ment</div>
Cks.....Casks.	Hlf.....Half.	
Co.....Company.	Hhd.....Hogshead.	
C. O. D..... <div style="display: inline-block; vertical-align: middle;">Collect on Delivery.</div>		

Pd..... Paid.	Rec't..... Receipt.	T..... Ton.
Per.....By, or by the.	R. R.....Railroad.	Trcs.....Tierces.
Pkgs.....Packages.	Rs., or rls.....Rolls.	Treas.....Treasurer.
Plts.....Plates.	Schr.....Schooner.	Ult.....Last month.
P. O.....Post Office.	Sh.....Ship.	Wt.....Weight.
Pr.....Pair.	Shipt.....Shipment.	Yds.....Yards.
Prox.....Next Month.	Stbt.....Steamboat.	Yr.....Year.
Ps., or pcs.....Pieces.	Stor.....Storage.	Viz.....Namely.
Qr.....Quarter.	Str.....Steamer.	Vs.....versus, or against.
Rec'd Pay't..... } Received } Payment.	Sunds.....Sundries.	
	Super.....Superfine.	

TIME ABBREVIATIONS.

Jan., or Jan'y.....January.	Aug.....August.	Tues.....Tuesday.
Feb., or Feb'y.....February.	Sept.....September.	Wed.....Wednesday.
Mar.....March.	Oct.....October.	Thurs.....Thursday.
Apr.....April.	Nov.....November.	Fri.....Friday.
May.....May.	Dec.....December.	Sat.....Saturday.
June, or Jun.....June.	Sun.....Sunday.	Wk.....Week.
July, or Jul.....July.	Mon.....Monday.	A.C.....Current year.

COMMERCIAL CHARACTERS.

@.....At.	(“).....Ditto, or the same.	℥.....per, or by the.
%.....Account.	%..... } per centum, or by } the hundred.	£.....Pounds sterling.
c.....Cents.	%.....New Account.	×.....By, as 3 × 4 inches.
%.....Care of.	℥.....Number.	1 ¹One and one-fourth.
✓.....Check mark.	%.....Old Account.	1 ²One and two-fourths.
\$.....Dollars.		1 ³One and three-fourths.

SAMPLES OF BILLS, STATEMENTS, ETC.

NOTE.—Let the student make neat and accurate pen-and-ink copies of the following sample business forms.

BILL OF SERVICES.

(RECEIPTED.)

Henderson, Mo., Dec. 21, 1894.

J. H. Wilson

To Henry Hart, Dr.

5 $\frac{1}{2}$ days' work clearing land, @ \$2.50, \$13.25

Received payment.

Henry Hart.

DENOMINATE NUMBERS.

RETAIL GROCER'S BILL.

Student will carry out extensions and find totals.

Eureka, Cal., July 10, 1895.

M J C Craddock

BOUGHT OF

BOUGHT OF MARSHALL & DINSMORE.

Dealers in Groceries and Provisions.

226 F STREET

Terms: 30 days.
Int. 1 per cent per month on all overdue accts.

June	1	Bal. as per Statement Rendered	3	15
	3	Flour ⁸⁵ Beans ²⁵ Butter ⁴⁰ 2* Coffee ³⁵ Meal ²⁵ Rice ³⁵ Bacon ⁴⁰ Ham ¹⁰ #		
	6	C. Meal ³⁰ Salt ¹⁰ N.O Mol ⁹⁵ Sugar ^{2.00} 25#		
	9	Broom ³⁵ W Tub ⁹⁷ C. Corn ^{1.25} 1 dz Pickles ^{1.95} Soda ^{1.20} Tomatoes ^{1.10} 1 dz Potatoes ^{1.20} 4 Bu		
	13	Eggs ³⁶ 2 dz Mackerel ⁷⁵ l C Oil ⁶³ 5 gal		
	19	S.S Syrup ⁶⁵ Olive ⁵⁰ 1 qr Rice ²⁵ Butter ³⁵ 2*		
	26	Flour ⁷⁵ 25* D Apples ⁴⁵ Cow Milk Beans ⁴⁵		
	15	Cr. by Cash	10	
	21	" " Syrup Returned Balance	65	

DENOMINATE NUMBERS.

INVOICE.

Quincy, Ill., Sept. 24, 1894.

M. R. S. Peters & Sons.Bought of THE RALF MILLS CO.,

MANUFACTURERS AND IMPORTERS OF

Pocket Books, Purses, Etc.

TERMS: 30 days, or 3 per cent disc. for Cash in 10 days.

$\frac{1}{4}$	dg	2078	8 ⁰⁰			
$\frac{1}{6}$	"	532	9 ⁰⁰			
$\frac{1}{6}$	"	275 B	13 ⁵⁰			
$\frac{1}{6}$	"	222 V	10 ⁵⁰			
$\frac{1}{6}$	"	20 V 19	10 ⁵⁰			
$\frac{1}{6}$	"	488	12 ⁰⁰			
$\frac{1}{6}$	"	219	10 ⁵⁰			
$\frac{1}{3}$	"	4864	4 ⁵⁰			
$\frac{1}{3}$	"	452	6 ⁰⁰			
$\frac{1}{2}$	"	227 B 15 A	16 ⁵⁰			
$\frac{1}{3}$	"	430	4 ⁰⁰			
$\frac{1}{6}$	"	248	6 ⁰⁰			
$\frac{1}{4}$	"	3037	6 ⁰⁰			
$\frac{1}{6}$	"	911	4 ⁰⁰			
$\frac{1}{4}$	"	4568	4 ⁰⁰			
$\frac{1}{4}$	"	951	4 ⁰⁰			
$\frac{1}{6}$	"	B 446	6 ⁰⁰			
$\frac{1}{4}$	"	B 428 - 2	4 ⁰⁰			
$\frac{1}{4}$	"	553 D	4 ⁰⁰			
$\frac{1}{6}$	"	290	6 ⁰⁰			
		Credit by Mdsv Returned				7 50
		Balance				

200. The student should make out in proper form the following Bills, Invoices, Statements, etc., according to the forms given in the models; make the extensions, and ascertain the totals.

COMMERCIAL EXERCISES.

1. On April 3, 1895, J. W. V. York, Detroit, Mich., bought of the Michigan Lumber Co. the following invoice of lumber:—

15140 ft. Pine Lumber,	\$13.75 per M., \$.....
32142 " Hemlock,	12.75 " "
14380 " Flooring,	22.00 " "
3485 " Scantling,	14.50 " "
9345 " Joists,	15.25 " "
12140 " Lumber Finished,	42.00 " "
14180 " Shingles,	3.25 " "
19185 " Planks,	13.50 " "
9684 " Culls,	9.60 " "
5680 " Cedar Posts,	18.00 " C.

2. C. E. Blake & Sons, Coal Dealers, Dayton, O., sold to the Ohio Pump Company on account as follows:—

Sept. 1, 1894,	1780 lbs. Coal,	\$6.00 per Ton, \$.....
" 15, "	4560 " " "	3.56 " "
Oct. 15, "	13140 " " "	6.25 " "
Nov. 1, "	14250 " " "	5.80 " "
Dec. 15, "	9854 " " "	6.50 " "
Jan. 18, 1895,	14954 " " "	3.25 " "
" 31, "	8365 " " "	6.75 " "
Feb. 15, "	1240 " " "	6.65 " "
" 22, "	13184 " " "	4.25 " "
" 28, "	4564 " " "	6.50 " "

Prepare statement for March 1, 1895.

3. Make out the following invoice of dry goods sold P. Hoffmaster, Elgin, Ill., by Marshall Field & Co., Chicago, Ill., on Mar. 14, 1895. Terms 30 days. Case № 324:—

10 pcs. Prints, 34¹, 36³, 34², 25¹, 27, 28³, 38¹, 43¹, 45, 38² yds.,
4¹/₂ per yard.

12 pcs. Gingham, 34^3 , 37^1 , 45^3 , 22^1 , 27, 28^2 , 37^2 , 48^3 , 36^1 , 25^2 , 29, 38^2 yds., $7\frac{1}{2}\text{¢}$ per yard.

16 pcs. Cashmere, 52^3 , 63^2 , 51^2 , 63^1 , 45^2 , 27^3 , 54^1 , 65^2 , 38^3 , 65^3 , 45^1 , 39^1 , 66, 64^3 , 67^1 , 57^2 yds., 52¢ per yard.

4. G. S. McKinley, Minneapolis, Minn., sold C. F. Bayard & Co. the following invoice of Hardware on April 15, 1895. Make out invoice:—

6 doz. Gimlets,	\$.87 $\frac{1}{2}$,	\$.....
15 " Garden Rakes,	3.20
5 $\frac{1}{2}$ " Steel Shovels,	8.50
14 " Doorsprings,	2.50
12 gro. Screws,	1.10
15 doz. 3 inch Bolts,	.75
250 lbs. 10d Nails,	.05 $\frac{1}{2}$
3 $\frac{1}{2}$ doz. Chisels,	8.00
3 " Hammers,	13.50

5. The Morgan Paper Co., Detroit, Mich., sold J. W. Warr, invoices with amounts and dates as follows:—

Folio 325.	Jan. 2, 1895,	\$ 75.80.
	" 15, "	126.75.
	" 30, "	148.56.
	Feb. 24, "	97.84.
	Mar. 3, "	136.92.
	" 17, "	194.37.
	" 29, "	207.08.

Make out a statement of Warr's account for April 1, 1895.

6. E. Murray bought of Sherwin & Ball, Mar. 1, 1895, 8 lbs. Rio Coffee, \$.38; 4 lbs. Silver Leaf Tea, \$.55; $8\frac{1}{2}$ doz. Eggs, \$.16; $5\frac{3}{4}$ lbs. Butter, \$.20; 25 lbs. D. G. Sugar, $$.04\frac{1}{2}$; 5 lbs. Mocha Coffee, \$.33; $4\frac{1}{2}$ lbs. Italian Prunes, \$.12; $\frac{1}{2}$ gal. Maple Syrup, \$1.25; $4\frac{3}{4}$ lbs. Cheese, \$.16; $\frac{3}{4}$ doz. Canned Tomatoes, \$1.80. Make out Murray's bill and receipt it.

7. E. Barnes bought of Mabley & Co., Apr. 5, 1895:— Case \times 228, 20 pieces Percales: 41^2 , 36^3 , 45^2 , 25^3 , 26^1 , 28^2 , 38^1 , 22^3 , 29^3 , 39^2 , 37, 28^1 ,

29^s, 36ⁱ, 22ⁱ, 39ⁱ, 28^s, 29^s, 31, 32^s, at 5 $\frac{3}{4}$ ¢. Case \times 158. 24 pieces Serge. 38^s, 34^s, 45^s, 36, 48ⁱ, 22ⁱ, 36^s, 37^s, 38, 42^s, 43ⁱ, 48^s, 45^s, 36^s, 38ⁱ, 29^s, 38^s, 29^s, 36ⁱ, 48^s, 41, 36, 37^s, 38^s, at 37 $\frac{1}{2}$ ¢. Make out invoice.

8. E. O. Hoffman bought of the Holter Hardware Co., Apr. 6, 1895. 6 doz. Handsaws at \$8.50; 5 doz. Axes at \$10.75; 4 doz. Jackplanes at \$13.50; 3 $\frac{1}{2}$ doz. Shovels at \$11.50; 2 $\frac{1}{2}$ doz. Screwdrivers at \$2.64; 50 lbs. 10d Nails at \$.02 $\frac{1}{4}$; $\frac{1}{2}$ doz. Lawnmowers at \$23.50; 88 lbs. Horseshoe nails at \$.08 $\frac{1}{4}$; 4 $\frac{1}{2}$ doz. 5 in. Bolts at \$1.80. Make out invoice.

9. On July 10th, 1895 Henry S. Carter & Sons, Retail Grocers, sold to A. W. Good, the following bill of goods: Coffee, \$1.50; Canned Corn, 75¢; 1 Ham, \$2.75; Flour \$1.35; Eggs, 35¢; Syrup, 80¢; Starch, 25¢; Oatmeal 20¢; Beans, 65¢; Tea, \$1.25; Crock of Butter, \$2.00. Good paid, Cash, \$2.50, and Butter amounting to \$1.85 was returned. Make out bill.

10. Noyes & Peters, Wholesale Dealers in General Hardware, Cincinnati, O., during the month of May, 1895, sold to James Hodge & Sons, Marysville, Ind., invoices as follows: 1st, \$122.50; 5th, \$281.45; 9th, \$69.40; 11th, \$173.15; 16th, \$92.20; 23d, \$17.40; 26th, \$27.42; 31st, \$59.28. The balance against Hodge & Sons on May 1st was \$147.60, and during the month they made payments as follows: 9th, Cash, \$275; 15th, Draft on R. Chase & Co., \$200; and on the 17th, Cash \$145. Make out statement to June 1.

FOREIGN MONEY.

201. For persons engaged in foreign trade, travelers and others, it is often necessary to find the equivalent of a given sum of the money of one country in that of another. To facilitate exchange, the various nations have mutually agreed upon **Ratios of Exchange**, showing the relative value of different monetary standards. Thus it has been agreed between the United States and Great Britain that the value of the British Standard, the Pound Sterling shall be \$4.8665 in American gold.

202. These ratios are based upon the amount of **pure gold or silver** used in the compared standard coins, *not* upon the weight of the coins.

203. Following is a table showing the **Value in American Gold** of the standard coins of different nations, also the subdivisions of these coins most commonly used in effecting exchanges.

TABLE OF FOREIGN MONETARY STANDARDS.

Country.	Unit of Currency.	Standard.	Value in U. S. Money.
Argentine Repub.	Peso of 100 centavos.....	Gold and Silver .	\$.965
Austria	Florin of 100 kreutzer.....	Silver371
Belgium.....	Franc of 100 centimes.....	Gold and Silver .	.193
Bolivia	Boliviano of 100 centavos....	Silver751
Brazil	Milreis of 1000 reis.....	Gold546
British America	Dollar of 100 cents.....	Gold	1.00
Chili.....	Peso of 100 centavos.....	Gold and Silver .	.912
Cuba.....	Peso of 100 centavos.....	Gold and Silver .	.932
Denmark.....	Crown of 100 öre.....	Gold268
Ecuador.....	Peso of 100 centavos.....	Silver751
Egypt.....	Piaster of 40 paras.....	Gold049
France.....	Franc of 100 centimes.....	Gold and Silver .	.193
Great Britain...	Pound Sterling of 20 shillings	Gold	4.8665
Greece.....	Drachma of 100 lepta	Gold and Silver .	.193
German Empire..	Mark of 100 pfennige.....	Gold238
Guatemala	Peso of 100 centavos.....	Silver88
Hayti.....	Gourde of 100 cents.....	Gold and Silver .	.965
Honduras.....	Peso of 100 centavos.....	Silver68
India.....	Rupee of 16 annas.....	Silver357
Italy	Lira of 100 centesimi.....	Gold and Silver .	.193
Japan	Yen of 100 sen.....	Silver81
Liberia.....	Dollar of 100 cents.....	Gold	1.00
Mexico	Dollar of 100 centavos.....	Silver816
Netherlands.....	Florin or Guilder of 100 cents	Gold and Silver .	.402
Nicaragua.....	Peso of 100 centavos	Silver68
Norway	Crown of 100 öre.....	Gold.....	.268
Peru	Sol of 100 centavos.....	Silver751
Portugal.....	Milreis of 1000 reis.....	Gold.....	1.08
Russia.....	Rouble of 100 copecks.....	Silver601
Sandwich Islands	Dollar of 100 cents.....	Gold.....	1.00
Spain.....	Peseta of 100 centimes.....	Gold and Silver .	.193
Sweden.....	Crown of 100 öre.....	Gold.....	.268
Switzerland.....	Franc of 100 centimes.....	Gold and Silver .	.193
Tripoli.....	Mahbub of 20 piasters.....	Silver677
Turkey.....	Piaster of 40 paras	Gold.....	.044
U. S. of Columbia	Peso of 100 centavos.....	Silver68
Venezuela.....	Bolivar of 20 centavos.....	Gold and Silver .	.193

204. As monetary exchanges with England, Canada, France, and Germany are of frequent occurrence in the United States, the full tables of monetary units used in these countries are given.

ENGLISH MONEY.

205. English, or **Sterling Money**, is the legal tender money of the British Empire.

TABLE.

4 farthings (far.)	= 1 penny (d.),	1 penny	= \$.02+
12 pence	= 1 shilling (s.),	1 shilling	= \$.24+
20 shillings	= 1 pound (£),	£1	= \$4.86+

SCALE.—4 - 12 - 20.

NOTE.—The British gold coins are the sovereign (= £1), half-sovereign (= 10 s.).

The silver coins are the florin (= 2 s.), shilling, sixpenny, fourpenny, and threepenny pieces.

The copper coins are the penny, halfpenny, and farthing pieces.

The guinea (= 21 s.), half-guinea (10 s. 6d.) gold, and the crown (= 5 s.), and the half-crown (= 2 s. 6 d.), are no longer coined but are still in circulation.

The paper money of Great Britain is issued by the Bank of England, a semi-public corporation. Its bills circulate extensively as money, and, through government support, are kept at par with gold.

FRENCH MONEY.

206. The money of France is a **Decimal Currency**. The **Unit** is the **Franc** (silver). Its commercial value in U. S. gold is \$.193, but in estimating duties its legal value, \$.186, is taken.

TABLE.

10 millimes (m.)	= 1 centime (c.)
10 centimes	= 1 decime (d.)
10 decimes	= 1 franc (fr.)

SCALE.—Decimal.

NOTE.—Centimes (pronounced sentimes) are, in trade, usually called "cents."

The terms decimes and millimes, like our dimes and mills, are not used in business calculation, but are written as decimal parts of a franc. Thus, 49 francs 7 decimes 3 centimes and 8 millimes would be written 49.738 fr.

GERMAN MONEY.

207. The standard monetary unit of the German Empire is the *Reichsmark*, and is divided into 100 parts called *pfennige* (pennies). The *Reichsmark* is equal to \$.2385, or slightly less than the value of an English shilling.

NOTE.—Other units are the silver *thaler* (pronounced taller), value 74.6 cents, and the *groschen* ($= 2\frac{1}{2}\text{¢}$).

CANADA MONEY.

208. The coinage system of Canada is virtually the same as that of the United States. The silver coins are of the same denomination and value as those of the United States. Canada has no gold coins other than those of Great Britain. Canadian and United States coins circulate interchangeably and quite freely in either country, especially near the boundary.

MEASURES OF WEIGHT.

TROY WEIGHT.

209. *Troy Weight* is used in weighing gold, silver, jewels, etc.

TABLE.

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

The standard unit of weight in the United States is the *Troy pound*, which contains 5760 grains.

AVOIRDUPOIS WEIGHT.

210. *Avoirdupois Weight* is used for all ordinary purposes of weighing.

TABLE.

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
20 hundredweight, or 2000 lbs.	= 1 ton (T.)

In some instances, as in all United States Custom Houses and in coal and iron mines, the long ton or 2240 lbs. is still used.

COMPARISON OF AVOIRDUPOIS AND TROY WEIGHTS.

7000 grains Troy = 1 lb. Avair.	437½ grains Troy = 1 oz. Avair.
5760 grains Troy = 1 lb. Troy.	480 grains Troy = 1 oz. Troy.

ADDITIONAL AVOIRDUPOIS WEIGHTS.

1 Keg, Nails.....	100 lbs.
1 Barrel, Flour.....	196 "
1 Barrel, Pork or Beef.....	200 "
1 Cental, Grain or Flour.....	100 "
1 Barrel, Salt.....	280 "

MEASURES OF EXTENSION.

LINEAR MEASURE.

211. Linear or Long Measure is used in measuring lengths and distances.

TABLE.

12 inches (in.) = 1 foot (ft.)	5½ yards or 16½ ft. = 1 rod (rd.)
3 feet = 1 yard (yd.)	320 rods = 1 mile (mi.)
1 mile = 1760 yards = 5280 feet.	

SURVEYORS' LINEAR MEASURE.

212. Linear Measure is used by Surveyors in measuring boundaries of land, etc.

TABLE.

7.92 inches = 1 link (l.)	80 chains = 1 mile (mi.).
100 links = 1 chain (ch.)	1 chain = 4 rods = 22 yards = 66 feet.

ADDITIONAL LINEAR MEASURES.

18 inches = 1 cubit.	5 paces = 1 rod.
3 " = 1 palm.	6 feet = 1 fathom.
4 " = 1 hand.	3 miles = 1 league.

MEASURES OF SURFACE.

SQUARE MEASURE.

213. Square Measure is used in measuring all surfaces, such as land, boards, plastering, painting, etc.

TABLE.

144 square inches (sq. in.)	= 1 square foot (sq. ft.)
9 square feet	= 1 square yard (sq. yd.)
30¼ square yards	= 1 square rod (sq. rd.)
160 square rods	= 1 acre (A.)
640 acres	= 1 square mile (sq. mi.)
1 acre	= 4840 square yards = 43560 sq. ft.

SURVEYORS' SQUARE MEASURE.

214. This measure is used by surveyors in measuring lands.

TABLE.

10000 square links (sq. l.)	= 1 square chain (sq. ch.)
10 sq. ch.	= 1 acre.
640 acres	= 1 sq. mi., or section.
36 sq. mi., or sections,	= 1 township.

MEASURES OF CAPACITY.

LIQUID MEASURE.

215. Liquid Measure is used in measuring liquids.

TABLE.

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
31½ gallons	= 1 barrel (bbl.)
63 gallons, or 2 barrels	= 1 hogshead (hhd.)

DRY MEASURE.

216. Dry Measure is used in measuring grain, fruit, vegetables, etc.

TABLE.

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

CUBIC MEASURE, OR MEASURES OF VOLUME.

217. Cubic Measure is used in measuring things which have length, breadth and thickness.

TABLE.

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
16 cubic feet	= 1 cord foot (cd. ft.)
8 cord feet, or 128 cu. ft.	= 1 cord
24½ cu. ft.	= 1 perch of masonry.

A cord of wood is a pile 8 ft. long, 4 ft. wide, and 4 ft. high.

CIRCULAR OR ANGULAR MEASURE.

218. Circular Measure is used in measuring angles and in estimating latitude and longitude.

219. A Degree, which is the $\frac{1}{360}$ part of the circumference of a circle, is the unit of measure.

220. An Angle is the opening between two lines which diverge from a common point.

221. A Circle is a plain figure bounded by a curved line, every point of which is equally distant from a point within, called the center.

222. The Circumference of a circle is the line that bounds it.

223. The Diameter of a circle is a straight line which passes through the center, and terminates at each end in the circumference.

224. The Radius of a circle is a straight line extending from the center to the circumference. The radius is $\frac{1}{2}$ of the diameter.

TABLE.

60 seconds (")	= 1 minute (')
60 minutes	= 1 degree (°)
360 degrees	= 1 circle (c.)

MEASURES OF TIME.

225. A Mean Solar Day is the standard measure of time and is the average time required by the earth to make one revolution on its axis.

TABLE.

60 seconds (sec.)	= 1 minute (min.)
60 minutes	= 1 hour (hr.)
24 hours	= 1 day (da.)
7 days	= 1 week (wk.)
4 weeks	= 1 lunar month.
365 days, or 12 months (mo.)	= 1 common year (yr.)
366 days	= 1 leap-year.
10 years	= 1 decade.
100 years	= 1 century.

M. (Meridian) denotes 12 o'clock midday.

A. M. (Ante Meridian) denotes the 12 hours before noon.

P. M. (Post Meridian) denotes the 12 hours after noon, or the time between noon and the following midnight.

226. In most business transactions, 30 days are considered a month, and 360 days a year.

227. A Solar Year is the time in which the earth makes one revolution around the sun, or 365 days 5 hours 48 minutes and 49.7 seconds, or $365\frac{1}{4}$ days nearly.

228. A Common Year consists of 365 days, and 4 common years are nearly 1 day less than 4 solar years; for this reason, to every fourth year, with the exception of the centennial years not divisible by 400, 1 day has been added in order to rectify this error. This extra day is added to the month of February, which then has 29 days.

Years divisible by 4, except the centennial years, are leap-years, and centennial years divisible by 400 are leap-years.

TABLE OF MONTHS.

1st, January, having 31 days.	7th, July, having 31 days.
2d, February, " 28 or 29 "	8th, August, " 31 "
3d, March, " 31 "	9th, September, " 30 "
4th, April, " 30 "	10th, October, " 31 "
5th, May, " 31 "	11th, November, " 30 "
6th, June, " 30 "	12th, December, " 31 "

A calendar month is the time from any date until the corresponding date in the following month. Thus, from February 15 to March 15 of the same year would be a calendar month.

26 working-days constitute a legal working-month, and 6 working-days a week.

The civil, or legal day, begins at midnight and continues until the following midnight.

In most states a legal day's work consists of 10 hours' work.

PAPER MEASURE.

TABLE.

24 sheets	= 1 quire (qr.)
20 quires	= 1 ream (rm.)
. 2 reams	= 1 bundle (bdl.)
5 bundles	= 1 bale (bl.)

COUNTING.

229. In counting, the following table is used :—

20 units = 1 score.
 12 units = 1 dozen (doz.)
 12 dozen = 1 gross (gro.)
 12 gross = 1 great gross (gr. gro.)

QUIZ FOR REVIEW.

230. The following Quiz will be found useful to the teacher in testing the student's memory of the more important parts of the tables. The answers may be oral or written.

- | | |
|--------------------------------|-----------------------------------|
| 1. Pks. in 1 bu.? | 26. Oz. in 1 lb. (Troy)? |
| 2. Rds. in 1 mi.? | 27. Val. in U. S. gold of 1 fr.? |
| 3. Deg. in a circle? | 28. Sq. ft. in 1 sq. yd.? |
| 4. Gr. in 1 lb. (Troy)? | 29. Gal. in 1 bbl.? |
| 5. Days in a leap-year? | 30. Value in U. S. gold of 1s.? |
| 6. Feet in a rod? | 31. Lbs. in a short ton? |
| 7. Oz. in 1 lb. (Avoir.)? | 32. Days in one working month? |
| 8. Ft. in a fathom? | 33. Leap years from 1887 to 1906? |
| 9. Mills in \$1? | 34. Gr. in 1 pwt.? |
| 10. Value in U. S. gold of £1? | 35. Cu. ft. in 1 perch? |
| 11. Ft. in 1 mi.? | 36. Cu. in. in 1 gal. (liquid)? |
| 12. Sheets in 1 quire? | 37. Shillings in £1? |
| 13. Cu. in. in 1 cu. ft.? | 38. Yds. in 1 mi.? |
| 14. Lbs. in 1 keg nails? | 39. Gills in 1 pt.? |
| 15. Gr. in 1 lb. (Avoir.)? | 40. In. in 1 yd.? |
| 16. Yds. in 1 rod? | 41. Lbs. in 1 long ton? |
| 17. Sq. ch. in 1 acre? | 42. Weeks in 1 yr. |
| 18. Things in 1 gross? | 43. Min. in 1 degree? |
| 19. Sq. rds. in 1 acre? | 44. Pence in 1 shilling? |
| 20. Qts. in 1 peck? | 45. Cu. ft. in 1 cord? |
| 21. Gal. in 1 hhd.? | 46. Qts. in 1 gal.? |
| 22. Acres in 1 sq. mi.? | 47. One reichsmark in U. S. gold? |
| 23. Quires in 1 ream? | 48. Name U. S. gold coins. |
| 24. Inches in 1 hand? | 49. Cu. ft. in 1 cu. yd.? |
| 25. Sq. yds. in 1 sq. rd.? | 50. Lbs. in 1 bbl. of flour? |

REDUCTION OF DENOMINATE NUMBERS.

231. Reduction of Denominate Numbers is the process of changing them to other numbers of equal value but having different measurement units.

DESCENDING REDUCTION.

232. Descending Reduction is the process of changing one denominate number to another expressed in lower measurement units.

ILLUSTRATIVE EXAMPLE.

Reduce 4 yds. 2 ft. 5 in. to inches.

SOLUTION.

yds.	ft.	in.
4	2	5
3		
<hr/>		
12	ft.	
2		
<hr/>		
14	ft.	
12		
<hr/>		
168	in.	
5	"	
<hr/>		
173	in.	Ans.

EXPLANATION.—In 4 yds. there are 4×3 ft., or 12 ft.; adding the 2 ft. we have 14 ft. 14×12 in. = 168 in., to which we add the 5 in., giving 173 in., the answer.

NOTE.—The additions can usually be performed *mentally*, and need not be shown in the solution.

EXAMPLES FOR DRILL.

Reduce—

1. 25¢ to mills.
2. \$25 15¢ to mills.
3. £12 15s. to shillings.
4. £25 4s. and 4d. to farthings.
5. 9 lbs. 8 oz. 5 pwt. to pennyweights.
6. 4 T. 5 cwt. 27 lb. to ounces.
7. 95 lb. Troy to grains.
8. 2 mi. 44 rds. 2 yds. to yards.
9. 5 rds. 4 yds. 2 ft. 8 in. to inches.
10. 2 mi. 3 ch. to links.
11. 16 ch. 3 rds. to rods.

12. 1 A. 140 sq. rds. to sq. yds.
13. 25 sq. yds. 2 sq. ft. 113 sq. in. to sq. in.
14. 3 cu. yds. 6 cu. ft. to cu. in.
15. 8 cords to cubic feet.
16. 15 gal. 2 pt. to pints.
17. 14 bu. 4 pk. to quarts.
18. How many sheets of paper in 25 reams?
19. How many articles in 15 gross?
20. If a man works $8\frac{1}{3}$ hours a day and is to receive $15\frac{1}{2}$ ¢ an hour, how much will he earn in 1 working-month?

ASCENDING REDUCTION.

233. Ascending Reduction is the process of changing one denominate number to another one expressed in higher measurement units.

ILLUSTRATIVE EXAMPLE.

Reduce 127 pints to higher denominations.

SOLUTION.

pt.
 $2 \overline{) 127}$
 $8 \overline{) 63}$ qt. 1 pt.
 $4 \overline{) 7}$ pk. 7 qt.
 1 bu. 3 pk.
 bu. pk. qt. pt.
 1 3 7 1 Ans.

EXPLANATION.—Dividing 127 pt. by 2 gives 63 qt., and 1 pt. remaining. Dividing 63 qt. by 8 gives 7 pk. and 7 qt. remaining. Dividing 7 pk. by 4 gives 1 bu. and 3 pk. remaining. Uniting these quantities we have 1 bu. 3 pk. 7 qt. 1 pt. as the answer.

EXAMPLES FOR DRILL.

Reduce to higher denominations:—

- | | |
|-----------------------------|-----------------------------------|
| 1. 6174 ft. | 9. 33,148 oz. |
| 2. 5420 pt. | 10. 56,000 gr. Troy to lb. Avoir. |
| 3. 5684 mills. | 11. 94,681 sheets of paper. |
| 4. 3614 inches. | |
| 5. 3164 cu. ft. | 12. 516,489 seconds. |
| 6. 1024 cord feet to cords. | 13. 1,348 feet. |
| 7. 9648 d. | 14. 5164 sq. in. |
| 8. 53164 sq. ft. | 15. 1648 gills. |

FRACTIONAL DENOMINATE NUMBERS.

234. It is frequently necessary to reduce fractional denominate numbers to lower or higher units.

ILLUSTRATIVE EXAMPLES

Reduce $\frac{5}{7}$ of a bu. to lower units.

SOLUTION.

$$\begin{array}{rcll} & \text{pk.} & \text{pk.} & \text{pk.} & \text{pk.} \\ \frac{5}{7} \times 4 & = & \frac{20}{7} & = & 2 + \frac{6}{7}. \\ & \text{qt.} & \text{qt.} & \text{qt.} & \text{qt.} \\ \frac{6}{7} \times 8 & = & \frac{48}{7} & = & 6 + \frac{6}{7}. \\ & \text{pt.} & \text{pt.} & & \\ \frac{6}{7} \times 2 & = & \frac{12}{7} & = & 1\frac{5}{7} \text{ pt.} \\ \text{pk.} & \text{qt.} & \text{pt.} & & \\ 2 & 6 & 1\frac{5}{7} & \text{Ans.} & \end{array}$$

EXPLANATION.—Proceed as in Descending Reduction of whole numbers, applying the principles of multiplication of fractions.

Reduce $\frac{11}{12}$ of a foot to rods.

SOLUTION.

$$\begin{array}{rcll} \text{ft.} & \text{ft.} & \text{yds.} & \\ \frac{11}{12} \div 3 & = & \frac{11}{36}. & \\ \text{yds.} & \text{yds.} & \text{rds.} & \\ \frac{11}{36} \div 5\frac{1}{2} & = & \frac{1}{18}. & \text{Ans.} \end{array}$$

EXPLANATION.—Reduce as in whole numbers, employing the principles of division of fractions.

EXAMPLES FOR DRILL.

Reduce to integers or fractions of lower denominations—

- | | |
|----------------------------|----------------------------|
| 1. $\frac{1}{4}$. | 6. $\frac{5}{11}$ acres. |
| 2. $\frac{2}{3}$ lb. Troy. | 7. $\frac{4}{15}$ bushels. |
| 3. $\frac{3}{4}$ tons. | 8. $\frac{3}{16}$ miles. |
| 4. $\frac{5}{6}$ days. | 9. $\frac{3}{5}$ gallons. |
| 5. $\frac{7}{8}$ pecks. | 10. $\frac{7}{8}$ cu. yd. |

Reduce—

- | | |
|---------------------------------------|--|
| 11. $\frac{5}{7}$ in. to yd. | 14. $\frac{7}{8}$ shillings to pounds. |
| 12. $\frac{7}{8}$ sq. rd. to acres. | 15. $\frac{11}{12}$ cu. ft. to cu. yd. |
| 13. $\frac{5}{6}$ oz. to lb. (Avoir.) | 16. $\frac{9}{14}$ oz. to lb. (Troy.) |

DENOMINATE DECIMALS.

235. To reduce denominate decimals to integers of lower denominations.

ILLUSTRATIVE EXAMPLE.

Reduce £.625 to lower denominations.

SOLUTION.

$$\begin{array}{r} \text{£.625} \\ 20 \\ \hline 12.500 \text{ s.} = 12 \text{ s. } 6 \text{ d.} \\ 12 \\ \hline 6 \text{ d.} \end{array}$$

EXPLANATION.—As £1 = 20 s. £.625 will contain .625 times 20 s. or 12.5 s.; set down 12 s. as part of the result required, and reduce the decimal part, .5 s., to the next lower denomination (d.) by multiplying by 12, obtaining 6 d.; therefore £.625 = 12 s. 6 d.

EXAMPLES FOR DRILL.

Reduce to lower denominations—

- | | |
|--------------|-----------------|
| 1. .375 yds. | 5. .36125 tons. |
| 2. .245 yds. | 6. .784 bbl. |
| 3. .785 bu. | 7. .3565 deg. |
| 4. .875 mi. | 8. .9875 lb. |

236. To reduce a compound denominate number to a decimal of a higher denomination.

ILLUSTRATIVE EXAMPLE.

Reduce 10 mo. 15 da. to the decimal of a year.

SOLUTION.

$$15 \div 30 = .5, \text{ then } 10 \text{ mo. } 15 \text{ da.} = 10.5 \text{ mo.}$$

$$10.5 \div 12 = .875 \text{ of a year.}$$

EXPLANATION.—Begin with the lowest denomination (days), and reduce to the next higher denomination (months) by dividing by the number of days in a month, 30; the result is .5; prefix the 10 months, as 10.5 months, and reduce this result to the next higher denomination (years) by dividing by 12, the number of months in a year; the result is .875 of a year.

EXAMPLES FOR DRILL.

1. Reduce 7 mo. 15 da. to the decimal of a year.
2. Reduce 3 pks. 2 qts. to the decimal of a bushel.
3. Reduce 4 yds. 1 ft. 6 in. to the decimal of a rod.

4. Reduce 150 rds. to the decimal of a mile.
5. Reduce 7 cord-feet to the decimal of a cord.
6. Reduce 2 cwt. 80 lb. to the decimal of a ton.
7. Reduce 15s. 6d. to the decimal of £1.
8. Reduce 12 hr. 30 min. to the decimal of a day.
9. Reduce 8 oz. 12 pwt. 12 gr. to the decimal of a pound.
10. Reduce 12 cu. ft. to the decimal of a cubic yard.

ADDITION OF COMPOUND NUMBERS.

237.

ILLUSTRATIVE EXAMPLE.

Add 4 bu. 3 pk. 2 qt., 5 bu. 2 pk. 4 qt., 6 bu. 3 pk. 6 qt.

SOLUTION.

bu.	pk.	qt.
4	3	2
5	2	4
6	3	6

17 bu. 1 pk. 4 qt.

EXPLANATION.—Arrange the numbers so that those of similar denominations shall fall in the same column; begin with the lowest denomination and add this column, which gives 12 qt.; reduce this number to the next higher denomination (peck) by dividing by 8, the number of quarts in a peck, which gives 1 pk. 4 qt.; write 4 qt. under the first column; then add the second column, and to this sum add the 1 pk. carried from the first column; this result is 9 pk.; reduce to bushels, which gives 2 bu. 1 pk.; write the 1 pk. under the second column; then add the last column and to the sum add the 2 bu. carried from the second column, and place the sum under the last column.

EXAMPLES FOR DRILL.

1.	bu.	pk.	qt.	pt.	2.	T.	cwt.	lb.	oz.	3.	£.	s.	d.
	3	1	1	1		4	10	25	4		15	4	3
	4	2	3			3	5	18	7		6	3	8
	5	3	2	1		17	8	4	12		9	18	8
	15	2	1	1		13	7	29	13		25	6	4
	12	3	2	1		9	15	52	8		96	16	6
	4	1	2	1		16	4	24	9		14	8	9

4. A coal-dealer sold 5 loads of coal of the following weights: first load 2 T. 478 lb.; second load 3 T. 1624 lb.; third load 1 T. 925 lb.; fourth load 3 T. 640 lb., fifth load 1 T. 1728 lb. What was the total weight of the coal?

5. A grocer bought 5 barrels of syrup; the first barrel contained 28 gal. 2 qt. 1 pt.; the second, 29 gal. 1 qt. 1 pt.; the third, 25 gal. 3 qt.; the fourth, 31 gal. 2 qt. 1 pt.; the fifth, 30 gal. 3 qt. 1 pt. Find the total quantity of syrup bought.

6. A man, in a week's travel, went 10 mi. 50 rds. 2 yds. 2 ft. 8 in. the first day; the second day, 3 mi. 40 rds. 4 yds. 1 ft. 9 in.; third day, 14 mi. 165 rds. 3 yds. 2 ft. 8 in.; fourth day, 15 mi. 120 rds. 4 yds.; fifth day, 16 mi. 55 rds. 3 yds. 2 ft. 6 in.; sixth day, 20 mi. 46 rds. 1 yd. 1 ft. 7 in. What was the entire distance traveled during the week?

SUBTRACTION OF COMPOUND NUMBERS.

238.

ILLUSTRATIVE EXAMPLE.

From £18 10s. 5d. take £10 12s. 4d.

SOLUTION.

£	s.	d.
18	10	6
10	12	4
£7	18s.	2d.

EXPLANATION.—Arrange the subtrahend and minuend so that numbers of like denominations will fall in the same column; begin with the lowest denomination, take 4d. from 6d. leaves 2d., and set the 2 down under the first column; then, as 12 s. cannot be taken from 10 s., borrow £1, or 20 s., then 12 s. from 10 s. + 20 s. leaves 18 s.; set this down under the second column; then, as £1 was borrowed from the £18 leaves £17, and 10 from 17 leaves £7.

EXAMPLES FOR DRILL.

Find the difference between —

1. lb.	oz.	pwt.	2. mi.	rd.	yds.	ft.	3. cu. yds.	cu. ft.	cu. in.
17	6	14	3	140	3	1	14	15	1224
11	8	13	1	75	2	2	5	17	1125

4. A man had 25 acres of land and sold 14 acres 125 sq. rds. 25 sq. yds. What has he remaining?

5. If a barrel of vinegar contained $31\frac{1}{2}$ gal., and there was sold 14 gal. 2 qt. 1 pt., what quantity remained?

6. An employe engages to work for one year, and works 240 days and 12 hours; considering 300 working-days to the year, what time has he yet to work?

SUBTRACTION OF DATES.

239.

ILLUSTRATIVE EXAMPLE.

Find the time from July 25, 1893, to Sept. 15, 1894.

SOLUTION.

yr. mo. da.

1894 9 15

1893 7 25

1 1 20. Ans.

EXPLANATION.—Write the later date as the minuend, the year 1894 means 1894 years since the Christian Era, and September 15 means the 9th month and the 15th day of the month. Write the earlier date as subtrahend and subtract.

NOTE 1.—Consider 1 month as 30 days.

NOTE 2.—Where the time is less than a year, it is usually computed by counting the exact number of days in each month between the respective dates.

ILLUSTRATIVE EXAMPLE.

Find the time from Mar. 13, 1891, to July 26, 1891.

SOLUTION.

18 days left in March to the 31st,

30 days in April,

31 days in May,

30 days in June,

26 days in July to the 26th.

135 days. Ans.

EXPLANATION.—From March 13 to the 31st is 18 days; then, April has 30, May 31, June 30, and to the 26th of July; the sum of all these is 135 days.

EXAMPLES FOR DRILL.

Find the time between the following dates:—

1. April 1, 1872 and July 20, 1882.
2. March 23, 1875 and August 8, 1883.
3. July 1, 1892 and Oct. 8, 1894.
4. Sept. 15, 1891 and Jan. 13, 1894.

Find the exact number of days between—

5. Aug. 1, 1889 and Sept. 10, 1889.
6. July 22, 1890 and Nov. 12, 1890.
7. Nov. 16, 1890 and Jan. 3, 1891.
8. Jan. 4, 1891 and July 31, 1891.

9. Sept. 12, 1891 and Feb. 22, 1892.
10. Feb. 12, 1892 and Oct. 1, 1892.
11. Oct. 25, 1892 and Mar. 2, 1893.
12. Feb. 13, 1893 and Aug. 25, 1893.

MULTIPLICATION OF COMPOUND NUMBERS.

240.

ILLUSTRATIVE EXAMPLE.

Multiply 5 bu. 6 pk. 2 qt. by 9.

SOLUTION.

bu.	pk.	qt.
5	6	2
		9
<hr/>		
59 bu.	0 pk.	2 qt.

EXPLANATION.—Write the multiplicand and place the multiplier under the last denomination; then 9 times 2 qt. = 18 qt. = 2 pk. and 2 qt.; write the 2 qt. as a part of the product and carry the 2 pk. to the next product; 9 times 6 pk. = 54 pk. + 2 pk. carried, = 56 pk. = 14 bu. and no pecks remaining; then

place a cipher as the second part of the product and carry the 14 bu. to the next product; 9 times 5 bu. = 45 bu. + 14 bu. carried, = 59 bu., the last part of the product.

EXAMPLES FOR DRILL.

Multiply—

1. £10 6s. 8d. by 8.
2. 8 gal. 2 qt. 1 pt. 2 gills by 7.
3. 13 da. 12 hr. 15 min. 12 sec. by 20.
4. 14 lb. 10 oz. 12 pwt. by 12.
5. 2 mi. 20 rds. 4 yds. 2 ft. by 25.
6. 3 sq. rds. 15 sq. yds. 4 sq. ft. by 32.
7. A grocer bought 25 bbl. of vinegar averaging 31 gal. 3 qt. 1 pt. to the barrel. What was the total quantity of vinegar bought?
8. A man traveled 14 mi. 40 rds. 4 yds. per day. How far did he travel in 28 days?
9. A grocer bought 12 firkins of butter, averaging 42 lb. 12 oz. per firkin. How much butter did he buy, and what did it cost at 22¢ per pound?
10. If a man dig 3 rds. 4 yds. 2 ft. 8 in. in a trench every day, for 15 days, what length of trench will be dug at the end of that time?

DIVISION OF COMPOUND NUMBERS.

241. To divide compound numbers by abstract numbers.

ILLUSTRATIVE EXAMPLE.

Divide £125 14s. 8d. by 8.

SOLUTION.

£	s.	d.	
8)125	14	8	
<hr/>			
£15	£5 rem.		
	20		
	<hr/>		
	100s.		
	14		
	<hr/>		
8)114			
	14s.	2s. rem.	
	12		
	<hr/>		
	24d.		
	8d.		
	<hr/>		
	8)32d.		
	4d.		

EXPLANATION.—Divide 125 by 8, and the quotient is 15 and £5 remaining; write the £15 as the first part of the quotient; then reduce the £5 remaining to shillings, add the 14s. and the sum is 114s.; divide 114 s. by 8 and the quotient is 14 and 2s. remaining; write the 14s. as the second part of the quotient; then reduce the 2s. remaining to pence, and add the 8 d., which gives 32d.; divide the 32d. by 8 and the quotient is 4d., which write as the third part of the quotient.

£15 14s. 4d. Ans.

EXAMPLES FOR DRILL.

Divide—

1. 23 hr. 18 min. 24 sec. by 8.
2. 4 T. 3 cwt. 16 lbs. by 6.
3. 38 hr. 16 min. 20 sec. by 4.
4. 24 cu. yds. 4 cu. ft. by 5.
5. 38 bu. 3 pk. 2 qt. by 7.
6. 46 sq. rds. 18 sq. yds. 8 sq. ft. 60 sq. in. by 12.
7. 2 hhd. 1 bbl. 17 gal. 3 qt. by 11.
8. 5 lbs. 6 oz. 18 pwt. 16 gr. by 7.
9. A grocer sold 334 lbs. 12 oz. of butter in 6 days. What was the average amount sold per day?
10. A real estate dealer has 186 sq. rds. of land which he divides into 9 lots of equal size. How much land will there be in each lot?

242. To divide one compound number by another.

ILLUSTRATIVE EXAMPLE.

Divide 15 yds. 2 ft. 6 in. by 2 yds. 2 ft. 8 in.

SOLUTION.

15 yds. 2 ft. 6 in. = 570 in.

2 yds. 2 ft. 8 in. = 104 in.

$570 \div 104 = 5\frac{25}{52}$.

EXPLANATION.—Reduce the dividend and divisor to the lowest denomination mentioned in either, and then divide.

EXAMPLES FOR DRILL.

Divide —

1. £25 8s. 6d. by £3 4s. 9d.

2. 4 mi. 125 rds. by 8 yds. 2 ft.

3. 25 bu. by 3 bu. 2 pk. 7 qt.

4. A train had 2500 miles to go; if the average distance traveled per hour was 25 mi. 220 rds. how long would it take the train to reach its destination?

MEASUREMENTS.

243. To Measure a quantity is to find the number of given units it contains.

SURFACE MEASUREMENTS.

244. A Surface is that which has length and breadth.

245. Plane Figures are definite portions of a flat surface, as, the square, the triangle, or the circle.

246. The Area of a figure is the number of surface measurement units it contains.

247. The Dimensions of a plane figure are its length and breadth. They are frequently expressed by the sign \times . Thus, "a board 5×2 ft.," means a board 5 ft. long and 2 ft. wide.

MEASUREMENT OF RECTANGLES.

248. A Rectangle is a figure bounded by four straight lines and whose angles are right angles.

249. To find the area when the dimensions are given.

ILLUSTRATIVE EXAMPLE.

Find the area of a rectangular board 4 in. long and 3 in. wide.

sq. in.	sq. in.	sq. in.	sq. in.
sq. in.			
sq. in.			

EXPLANATION.—Let the accompanying figure represent the board. The board may be considered to consist of 3 rows of squares each row containing 4 sq. in., or four rows, each containing 3 sq. in. The area is therefore 12 sq. in.

RULE.—Multiply the length by the breadth, expressed in units of the same denomination; the result is the area.

EXAMPLES FOR DRILL.

1. Find the area of a room 18×20 feet.
2. How many square yards in a lawn, 65×46 ft.?
3. Find the area in square feet of a board 4 ft. 6 in. \times 5 ft. 4 in.

SOLUTION.

$4 \text{ ft. } 6 \text{ in.} \times 5 \text{ ft. } 4 \text{ in.} = 4\frac{1}{2} \text{ ft.} \times 5\frac{1}{3} \text{ ft.} = \frac{9}{2} \text{ ft.} \times 1\frac{2}{3} \text{ ft.} = 24$
sq. ft. Ans.

4. How many square feet in a floor 10 ft. 8 in. \times 11 ft. 3 in.?
5. How many square yards in a ceiling 13 ft. 6 in. \times 10 ft. 8 in.?

SOLUTION.

$$(13\frac{1}{2} \text{ ft.} \times 32\frac{2}{3} \text{ ft.} \times \frac{1}{9}) = \frac{27 \times 32}{2 \times 3 \times 9} = 16 \text{ sq. yds. Ans.}$$

NOTE.—The number of square yards $= \frac{1}{9}$ of the number of square feet. Always employ Compound Division or Cancellation in solving problems of this class.

6. How many square yards in a curtain 8 ft. 8 in. \times 22 ft. 6 in.?

7. How many acres in a field 94 rds. \times $62\frac{1}{2}$ rds.? (Answer in decimals, correct to 100ths.)

8. Find the square yards in the walls and ceiling of a room 24 ft. long, 18 ft. wide, and 10 ft. high.

SUGGESTION.—Find the linear distance around the room (24 ft. + 18 ft.) \times 2 = 84 ft.; $84 \times 10 = 840$ sq. ft., area of walls. Add to this the area of the ceiling ($24 \times 18 = 432$ sq. ft.), and divide by 9.

9. How many square yards in the walls and ceiling of a room 16 ft. 8 in. long, 12 ft. 6 in. wide, and 10 ft. high?

10. How many square feet in the entire surface of a block 7 ft. 2 in. long, 3 ft. 4 in. wide, and 2 ft. high?

APPLICATION OF SURFACE MEASURE.

CARPETING, PLASTERING, AND ROOFING.

250. Carpeting, and other woven goods of varying widths, are sold by the "running" or linear yard. Estimates are usually made by dividing the surface to be covered by the surface in one yard of goods.

ILLUSTRATIVE EXAMPLE.

How many yards of carpeting 28 in. wide will be required for a room 16 ft. 6 in. long and 14 ft. 8 in. wide?

SOLUTION.

$$16 \text{ ft. } 6 \text{ in.} = \frac{33}{2} \text{ ft.} \quad 14 \text{ ft. } 8 \text{ in.} = \frac{44}{3} \text{ ft.} \quad 28 \text{ in.} = \frac{7}{3} \text{ ft.}$$

$$\frac{33}{2} \times \frac{44}{3} \div (3 \times \frac{7}{3}) = \frac{33}{2} \times \frac{44}{3} \times \frac{1}{3} \times \frac{3}{7} = \frac{484}{14} = 34\frac{4}{7} \text{ yds.} \quad \text{Ans.}$$

EXPLANATION.—The product of the dimensions of the room divided by the product of the dimensions of one yard of carpet equals the number of yards required. Employ Compound Division, inverting the *whole divisor*.

251. It is sometimes necessary to lay the carpet in a certain direction, either lengthwise or crosswise of the room, and in order to allow for "matching," it may be necessary to buy more carpeting than enough to cover the room. In this case, use the "strip method."

ILLUSTRATIVE EXAMPLE.

How many yards of carpet 30 in. wide, laid lengthwise of the room, will be required for a room 18 ft. long and 14 ft. 4 in. wide?

SOLUTION.

$$14 \text{ ft. } 4 \text{ in.} = \frac{43}{3} \text{ ft.} \quad 30 \text{ in.} = \frac{5}{2} \text{ ft.}$$

$$\frac{43}{3} \text{ ft.} \div \frac{5}{2} \text{ ft.} = 5\frac{11}{5}, \text{ or } 6 \text{ strips.} \quad \frac{6 \times 18}{3} = 36 \text{ yds.} \quad \text{Ans.}$$

EXPLANATION.—Dividing the width of the room, $\frac{43}{3}$ ft., by the width of the carpet, $\frac{5}{2}$ ft., the quotient is $5\frac{11}{5}$. Hence, 6 strips of carpet, each 18 ft. long, or 108 ft., = 36 yds., will be required.

EXAMPLES FOR DRILL.

1. At \$1.75 per yard, what will be the cost of carpeting a room 21 ft. 4 in. long, 16 ft. 3 in. wide, with carpet 32 in. wide? No allowance for matching.

2. How many yards of carpeting, 27 in. wide and running crosswise of the room, will be required for a room 18 ft. 3 in. long and 15 ft. 6 in. wide?

3. At \$2.50 per yard, what will be the cost of carpeting a room 18 ft. 6 in. \times 21 ft. 3 in., with material 30 in. wide, allowing for waste in laying the carpet either lengthwise or crosswise according to the way that will require the least amount of carpet?

4. At 30¢ per square yard, what will be the cost of plastering the walls and ceiling of a room 17 ft. 6 in. long, 14 ft. 4 in. wide, and 12 ft. high, allowing for *one half* the surface in three doors, each $8 \times 4\frac{1}{2}$ ft.; and 4 windows, each $7\frac{1}{2} \times 3\frac{1}{2}$ ft.?

5. At \$2.75 per M. what will be the cost of shingles for a roof 28 ft. long and 16 ft. from eaves to comb, estimating one thousand shingles per square?

NOTE.—A Square in artificers' measurements, is a space 10×10 ft., or 100 sq. ft.

6. At \$15 per square, what will be the cost of slating a flat roof 62 ft. \times 40 ft. 6 in.?

7. At 8¢ per yard of material 30 in. wide, what will be the cost of cloth for a curtain 26 ft. long and 8 ft. 6 in. wide?

PAPERING.

252. Wall paper is sold in rolls of about 4 sq yds. As there is loss in matching and cutting for irregular spaces, accurate estimates are impracticable. Paper-hangers use the following

RULE.—*From the gross area in square feet of the walls and ceiling of the room, subtract 20 sq. ft. for each door or window. Divide the remainder by 33 ft. for the number of rolls.*

ILLUSTRATIVE EXAMPLE.

At 20¢ per roll, what cost the paper for a room 20 ft. long, 16 ft. wide, and 12 ft. high, the room having 3 doors and 5 windows?

SOLUTION.

$$(20 + 16) \times 2 = 72 \text{ ft. around the room.}$$

$$72 \times 12 = 864 \text{ sq. ft. in walls.}$$

$$20 \times 16 = 320 \text{ sq. ft. in ceiling.}$$

$$864 + 320 = 1184 \text{ sq. ft. in room.}$$

$$5 + 3 = 8 \text{ openings. } 8 \times 20 = 160 \text{ sq. ft.}$$

$$1184 - 160 = 1024 \text{ sq. ft. to be papered.}$$

$$1024 \div 33 = 31+, \text{ or } 32 \text{ rolls.}$$

$$32 \times 20 = \$6.40 \text{ cost.}$$

NOTE.—Fractional parts of a roll are not sold.

EXAMPLES FOR DRILL.

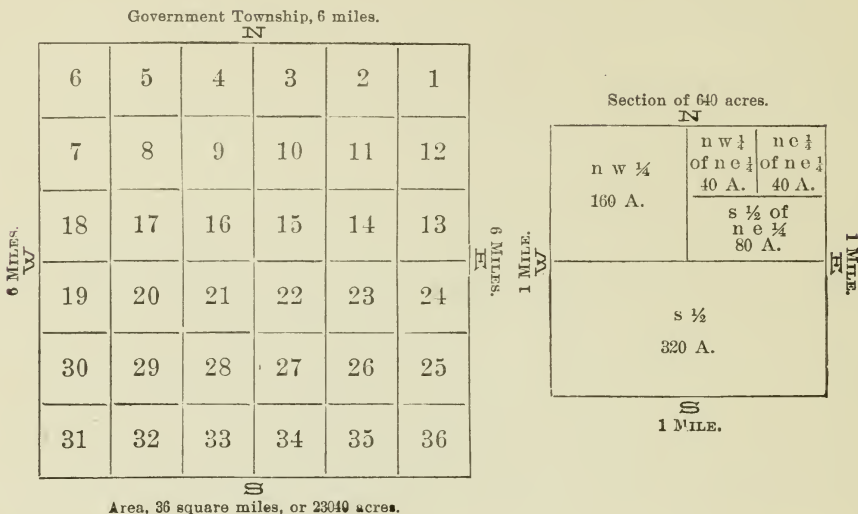
1. At 45¢ a roll, what cost the paper for a room 26 ft. long, 18 ft. 6 in. wide, and 13 ft. high, having seven windows and four doors?

2. At 30¢ per roll, what cost the paper for a room 15 ft. 4 in. \times 12 ft. 6 in., and 12 ft. high, including bordering at 7¢ per yard, to run entirely around the room?

LAND MEASURE.

253. United States Government Land is originally surveyed into *Townships*, each 6×6 miles. These are surveyed into *Sections* of one square mile, containing 640 acres, and uniformly numbered. These again, are surveyed into quarter-sections of 160 acres each, and these into halves or "eighties" of 80 acres each, quarters, or "forties," of 40 acres each, etc.

The numbering of sections in a township and the designation of fractional parts of sections is illustrated in the following diagram.



254. In estimating the area of land, surveyors use a measure called a **chain**, which is 4 rds. long, and divided for convenience of calculation, into 100 **links**. In a **square chain** there are 16 sq. rds.; and in 160 sq. rds., or 1 acre, there are ten square chains; hence to reduce square chains to acres, move the decimal point one place to the left.

ILLUSTRATIVE EXAMPLE.

How many acres in a lot 65 ch. 20 l. long and 40 ch. 15 l. wide?

SOLUTION.

$$40 \text{ ch. } 15 \text{ l.} = 40.15 \text{ ch.}$$

$$65 \text{ " } 20 \text{ " } = 65.2 \text{ "}$$

$$40.15 \times 65.2 = 2617.78 \text{ sq. ch.} = 261.778 \text{ acres.} \quad \text{Ans.}$$

PROBLEMS.

1. What is the value of the s e $\frac{1}{4}$ of sec. 11, at \$56 per acre?
2. A tract of land contains 4 townships and 23 sections. How many acres in the tract?

3. A piece of land is 128 ch. \times 74 ch. 17 l. Find the number of acres.

4. At \$40 per acre, find the value of a lot 94 ch. 80 l. long and 56 ch. 38 l. wide.

5. At \$2.15 per rod what will it cost to fence the s $\frac{1}{2}$ of s e $\frac{1}{4}$ of sec. 13?

6. A piece of land 29 ch. 16 l. long and 15 ch. 20 l. wide was bought for \$1250, fenced at a cost of \$1.40 per rod, ploughed at a cost of \$1.10 per acre, and sold at \$50 an acre. What was the gain or loss?

255. To find one dimension of a rectangle when the area and the other dimension are given.

ILLUSTRATIVE EXAMPLE.

A floor contains 680 sq. ft. and is 20 ft. wide ; required the length.

SOLUTION.

$680 \div 20 = 34$ ft. Ans.

EXPLANATION.—Since the area, 680 sq. ft., is the product of the dimensions, this number divided by 20, one of these factors, gives 34, the required factor.

RULE.—*Divide the area by the given dimension.*

NOTE.—It is necessary that the area and the given dimension should be corresponding units; thus, if the area is square yards, the given dimension should be linear yards.

PROBLEMS.

NOTE.—Carry inexact divisions to two decimal places.

1. A piece of land is 72 rds. long and contains 27 acres. What is its width in rods?

2. What should be the length in feet of a side-walk 12 ft. wide, in order that it may contain 96 sq. yds.?

3. A roadway is 2 rds. wide and contains 20 acres. What is its length in rods?

4. Find the width in chains and links, of a piece of land, 60 ch. 15 l. long, that contains 160 acres.

MEASUREMENT OF THE TRIANGLE.

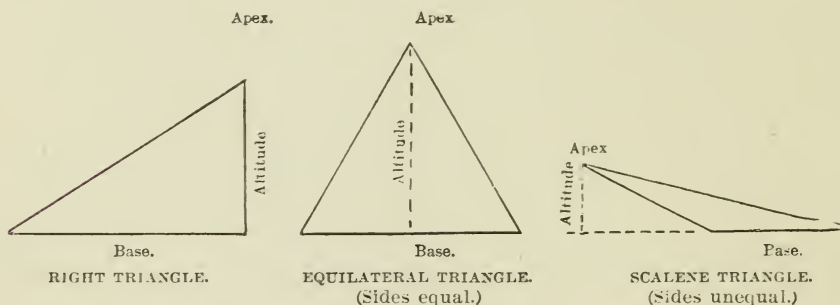
256. A Triangle is a plane figure having three straight sides.

257. The Base of a triangle is the side upon which it is supposed to rest. Either side may be assumed as the base.

258. The Apex of a triangle is the point opposite the base.

259. The Altitude of a triangle is the perpendicular distance from the apex to the plane, upon which the base is supposed to rest.

DIAGRAMS.



RULE.— *The Area of a triangle is equal to one-half the product of its base and altitude.*

ILLUSTRATIVE EXAMPLE.

The base of a triangle is 12 ft. and the altitude 16 ft. Find the area.

$$\frac{12 \times 16}{2} = 96 \text{ sq. ft. Ans.}$$

PROBLEMS.

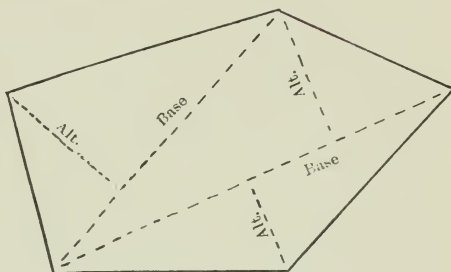
1. It is 75 rds. in a direct line from one corner of a triangular field to the opposite side, and the length of the latter is 90 rds. How many acres in the field?

2. At 20¢ per square yard, what will be the cost of painting the gable-end of a house 24 ft. wide, the comb of the roof being 8 ft. higher than the eaves?

3. At 15¢ per yard find the cost of the canvas for a wedge tent, 10 ft. square, and 8 ft. high, the sides being 9 ft. 4 in. to the ridge-pole, canvas 30 in. wide.

IRREGULAR FIGURES.

260. It is often desirable to measure irregular figures of various forms. Such figures are easily measured by dividing them into triangles, as in the accompanying diagram.

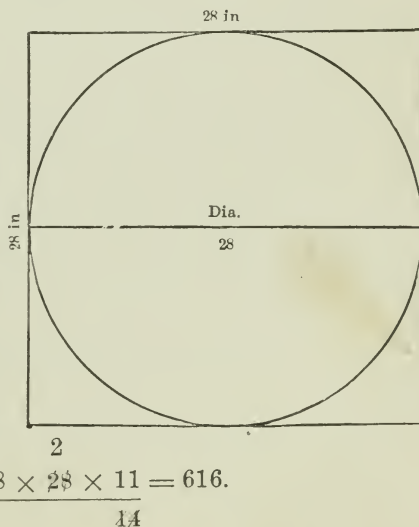


MEASUREMENT OF THE CIRCLE.

261. The area of any circle is practically $\frac{11}{14}$ (more accurately .7854) of the area of the square enclosing it. Examine the following diagram.

Area of square, $28 \times 28 = 784$ sq. in.
Area of circle, $784 \times \frac{11}{14} = 616$ sq. in.

If the diameter of the accompanying circle is 28 in., the enclosing square will evidently contain 28×28 or 784 sq. in. It has been proven by Geometry, that the circle in this case will contain very nearly $\frac{11}{14} \times 784$ sq. in., or 616 sq. in. In practical work it is unnecessary to get the area of the square: simply multiply the dimensions by $\frac{11}{14}$, thus $28 \times 28 \times \frac{11}{14} = 616$.



262. The Circumference of a circle is practically $3\frac{1}{7}$ (accurately 3.1416+) times the diameter. Hence either may be easily calculated when the other is given.

PROBLEMS.

1. A horse is tied to a rope 8 rds. long. Over how many square rods can he graze?
2. A wheel is $4\frac{1}{2}$ ft. in diameter. How many yards will it go in making 84 revolutions?
3. A cistern is $5\frac{1}{2}$ ft. in diameter. What is the area of the bottom in square feet?
4. How many square feet of sheet-iron will be required to make 200 ft. of 6 in. stove-pipe, allowing $\frac{3}{4}$ of an inch for seams?

SUGGESTION.—If the pipe be cut open lengthwise, the iron in it will form a rectangle, one side of which is the circumference of the pipe $+\frac{3}{4}$ in., and the other side the length of the pipe.

5. How many acres in a circular park 140 rds. in diameter?
6. At 7¢ per square foot, what will be the cost of the tin for the bottom and sides of a milk-can 28 in. in diameter and 4 ft. high?
7. What is the diameter in rods of a circular park one mile in circumference?
8. A bicyclist finds that his wheel, which is 28 in. in diameter, revolves 420 times per minute. What is his rate of speed per mile?
9. How many times will a wagon wheel 3 ft. 8 in. in diameter, revolve in running 5 miles?
10. How many square feet of tin will be required for 2 gross of tin pails, without lids, each 7 in. in diameter and 9 in. high?

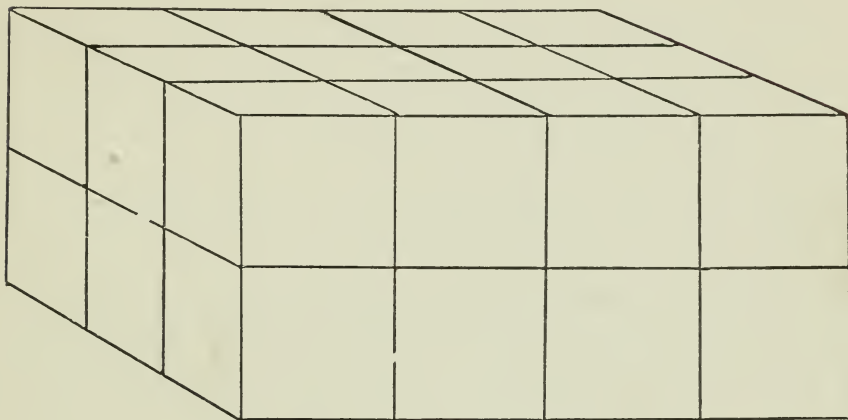
MEASUREMENT OF VOLUMES.

263. A Solid has length, breadth, and thickness.
264. The Volume of a solid is the amount of space it occupies, expressed in solid measurement units.
265. A Rectangular Solid is one that is bounded by rectangles.
266. The Dimensions of a rectangular solid are its length, breadth, and thickness.

ILLUSTRATIVE EXAMPLE.

What is the volume in cubic feet of a rectangular block 4 ft. long, 3 ft. thick, and 2 ft. high?

DIAGRAM.



EXPLANATION.—The area in square feet of the top or bottom of the block is 4×3 , or 12 sq. ft.; if it were 1 ft. high the volume would be 12 cu. ft.; but being 2 ft. high it occupies 2×12 , or 24 cu. ft.

PROBLEMS.

1. Find the volume in cubic yards of the earth taken from a ditch 65 ft. \times 3 ft. \times 5 ft.
2. How many cubic feet in a block 5 ft. 6 in. \times 2 ft. 8 in. \times 4 ft. 6 in.?

SUGGESTION.—Change the inches to fractional parts of a foot; the problem will then be $5\frac{1}{2} \times 2\frac{2}{3} \times 4\frac{1}{2} = (?)$ cu. ft.

3. How many cubic feet in a pile of stone 40 ft. 8 in. long, 6 ft. high; and 6 ft. 9 in. wide?
4. How many cubic yards will be removed in excavating a cellar 21 ft. 5 in. long, 18 ft. 4 in. wide and 6 ft. 9 in. deep?
5. How many cubic feet in a saw-log 28 in. in diameter and 24 ft long?

SUGGESTION.—Find the area of the end of the log and multiply by the length, or what is the same thing use the formula: $\frac{7 \times 7 \times 11 \times 24}{3 \quad 3 \quad 14}$.

LUMBER MEASUREMENT.

267. Lumber is sold by the **Board Foot** (bd. ft.), which is a board 1 ft. square and 1 in. thick, and contains 144 cu. in. or $\frac{1}{12}$ of a cubic foot of timber.

To find the board feet in any piece of timber: (1) *Multiply the area of one side in square feet by the thickness in inches*, or (2) *Multiply the cubic feet in the timber by 12*, or (3) *Multiply together any two dimensions in feet, and the remaining dimension in inches*.

268. Lumber is usually invoiced by stating the number of pieces, length in feet, and width and thickness in inches.

pcs. 1.

Thus, 18 — 24 — 6 × 4 means 18 pieces each 24 ft. long, 6 in. wide and 4 in. thick. When the dimensions are thus given, get the board feet by this formula:—

$$\frac{18 \times 24 \times 6 \times 4}{12} = 864 \text{ bd. ft.}$$

NOTE.—Dividing by the 12 is equivalent to changing one of the inch dimensions to feet. In lumber

1 in. thick, the board feet equal the square feet in the surface. Calculations can often be made mentally; do this whenever practicable.

PROBLEMS.

1. At \$2.50 per C. board feet, find the value of the following bill of lumber:—

pcs. 1.

28 18 . 8 × 2.

42 16 12 × 1.

68 18 3 × 2.

45 14 16 × 1.

NOTE.—In making the extensions, fractional portions of a board foot, if less than $\frac{1}{2}$, may be dropped; if greater than $\frac{1}{2}$, consider them as 1 bd. ft.

2. At \$13.60 per M., what will be the cost of the lumber for a side-walk 8 ft. wide and 180 ft. long, the planks to be 2 in. thick and to rest on 3 stringers 4 x 6 in.?

SUGGESTION.—In such problems, carpenters usually estimate the number of board feet required for each “running foot” of the walk, and multiply this result by the whole number of running feet, or total length of the walk.

3. How many board feet in a timber 48 ft. \times 3 ft. \times 2 ft.?
4. At \$24 per M. what will be the cost of lumber for a circular tank 16 ft. in diameter and 14 ft. high, the bottom to be of 3 in. and the sides of 2 in. planks?
5. At \$12.50 per M. for lumber and \$5.60 per C. for posts, what will it cost to fence a lot 40 rds. wide and 60 rds. long, allowing 5 bds. 8 \times 1 to the panel, the posts to be 10 ft. apart?

STONE WORK.

269. Stone Work is ordinarily estimated by the perch, of $24\frac{3}{4}$ cu. ft., of which 22 ft. is allowed for the stone, and $2\frac{3}{4}$ ft. for the mortar and filling.

No allowance is usually made in stone work for openings less than 3 ft. wide, nor for corners, unless in a close estimate of the material required.

ILLUSTRATIVE EXAMPLE.

At \$4.50 a perch, what is the cost of erecting the stone walls of a house, which is 40 ft. long, 30 ft. wide, the walls being 24 ft. high and 2 ft. thick, no allowance being made for openings?

SOLUTION.

$$\begin{aligned}
 (40 + 30) \times 2 &= 140 \text{ ft. outside measurement of walls.} \\
 140 \times 24 \times 2 &= 6720 \text{ cu. ft. in the walls.} \\
 6720 \div 24\frac{3}{4} &= 271\frac{17}{8} \text{ perches of stone work.} \\
 271\frac{17}{8} \times \$4.50 &= \$1221.82.
 \end{aligned}$$

RULE.—Find the number of cubic feet of masonry, and divide the result by $24\frac{3}{4}$, and the quotient will be the number of perches.

EXAMPLES FOR DRILL.

1. What is the cost of a stone foundation 40 ft. by 30 ft., 6 ft. high and 2 ft. thick, at \$4.95 a perch?
2. Cost of building a stone wall 200 ft. long, 5 ft. high and 4 ft. thick, at \$6.25 a perch?
3. At \$4.95 a perch, what is the cost of erecting the stone walls of a church 50 ft. long, 36 ft. wide, the walls being 25 ft. high and 2 ft. 3 in. thick; allowance being made for 1 door, 6 ft. 6 in. by 8 ft., and 4 windows 4 ft. by 8 ft.?

MEASUREMENT OF CAPACITIES.

270. In order to ascertain the capacity of bins, cisterns, boxes, granaries, etc., special terms and measures are used, of which the following will be found the most important.

271. Even, or Stricken Measure is used in measuring grains, small fruits, etc., and means the measuring vessel even full.

The stricken bushel contains 2150.42 cu. in.

272. Heaped Measure is used in measuring root-crops, vegetables, large fruits, etc.

The heaped bushel contains 2747.71 cu. in.

In practice 4 heaped bushels are considered equal to 5 stricken bushels.

A cubic foot is $\frac{4}{5}$, or .8, of a bushel, stricken measure, and .63 of a bushel, heaped measure. This is not exact, but close enough for all practical purposes.

The liquid gallon contains 231 cu. in. There are about $7\frac{1}{2}$ gal. in 1 cu. ft.

The gallon, dry measure, contains 268.8 cu. in.

The imperial bushel of Great Britain contains 2218.192 cu. in. and is equal to 1.03 standard bushels of the United States.

A cubic foot equals $\frac{5}{8}$ of a barrel (nearly).

In ordinary calculation of capacities, use dimensions and units in feet.

ILLUSTRATIVE EXAMPLES.

Find the capacity, in stricken bushels, of a wagon-bed 10 ft. 8 in. long, 2 ft. 3 in. deep, and 3 ft. 9 in. wide.

SOLUTION.

$$10 \text{ ft. } 8 \text{ in.} = 10\frac{2}{3} \text{ ft.}$$

$$2 \text{ " } 3 \text{ " } = 2\frac{1}{4} \text{ "}$$

$$3 \text{ " } 9 \text{ " } = 3\frac{3}{4} \text{ "}$$

$$8 \quad 5$$

$$\frac{32}{3} \times \frac{15}{4} \times \frac{9}{4} \times \frac{4}{5} = 72 \text{ bu.} \text{ Ans.}$$

EXPLANATION.—Multiplying together the dimensions in feet would give cubic feet, these by $\frac{4}{5}$ gives bushels.

Find the capacity in barrels of a circular tank 14 ft. in diameter and 27 ft. high.

SOLUTION.

$$\frac{14 \times 14 \times 11 \times 27 \times 5}{14 \times 21} = 990 \text{ bbls. Ans.}$$

EXPLANATION.—Squaring the diameter and multiplying by $1\frac{1}{4}$ gives the area of the bottom of the tank; this by 27 gives cubic feet, and

this by $\frac{5}{21}$ gives barrels. Unite these operations and cancel.

EXAMPLES FOR DRILL.

1. A wagon-body is 10 ft. long, 4 ft. wide, and 18 in. deep. How many bushels of potatoes will it hold?

2. How many bushels of oats will be required to fill a rectangular bin, 8 ft. long, 5 ft. wide, and 4 ft. deep?

3. A rectangular bin is 8 ft. 6 in. long, 5 ft. 6 in. wide, and 4 ft. 9 in. deep. How many bushels of wheat will it hold approximately?

4. A rectangular cistern is 6 ft. square, and 10 ft. deep. How many gallons of water will it hold?

5. A farmer has a rectangular bin 9 ft. long, 5 ft. 6 in. wide, and 4 ft. deep, which is full of wheat which he sells at 52¢ a bushel. What did he get for it? (Practical Rule.)

6. How many gallons of water in a well, that is 5 ft. in diameter, if the water is 38 ft. deep?

7. How many barrels of water will a cistern hold, that is 10 ft. deep and 5 ft. in diameter?

8. What must be the depth of a cistern 6 ft. in diameter, to hold 1500 gallons of water?

9. A wagon-body is 4 ft. wide and 18 in. deep. How long must it be, approximately, to hold 40 bu. of potatoes?

10. How many gallons of water will a reservoir hold, which is 100 ft. in diameter and 20 ft. deep?

11. A rectangular bin 12 ft. long and 6 ft. wide, will hold exactly 300 bu. of wheat. How high is the bin?

12. If a cistern is 6 ft. in diameter, how deep must it be to hold 100 bbls. of water?

BOOK MEASURE.

273. The pages of an ordinary book are printed upon sheets of paper of a standard size, being 17 in. wide and 22 in. long, and called "folio sheets."

274. If the leaves of a book are formed by folding such a sheet once, the book is called folio. Such a book is of the largest standard size, has four pages to each sheet, and each sheet is 17 in. long and 11 in. wide.

275. The following table shows the number of leaves and pages to the sheet, and the sizes of the pages in standard books:—

Size of book.	Leaves to each sheet.	Pages to each sheet.	Length of page.	Width of page.
Folio.....	2	4	17 in. ...	11 in.
Quarto.....	4	8	11 " ...	8½ "
Octavo.....	8	16	8½ " ...	5½ "
16mo.	16	32	5½ " ...	4½ "
12mo.	12	24	7½ " ...	4½ "

1. At 8¢ per quire what will be the cost of paper for an edition of 5000 copies of an octavo book containing 360 pages?

SOLUTION.

$$\begin{array}{r} 2500 \quad 15 \\ 5000 \times 360 \times \$ \\ \hline 16 \times 24 \\ 2 \end{array} = \$375.00 \text{ Ans.}$$

EXPLANATION.—5000 × 360 equals the total number of pages; this divided by 16, the pages to the sheet in an octavo book, gives the sheets; this divided by 24 gives the quires, and this multi-

plied by 8 gives the cost, \$375. Unite the operations and cancel.

PROBLEMS.

1. At 7¢ per pound what will cost the paper for an edition of 20000 copies of a quarto book of 280 pages, paper weighing 90 lbs. to the ream?

2. At \$14.75 per ream, what costs the paper for a 10000 copy edition of a duodecimo book of 320 pages?

3. At 11½¢ per pound what cost the paper for a 15000 copy edition of an octavo book of 400 pages, printed on 70 lb. paper (70 lb. to the ream)?

WATER PRESSURE.

276. Water and other liquids, exert **Pressure** upon the bottom and sides of vessels containing them. Pressure varies with the depth, and on any surface is equal to the weight of a column of the liquid extending from that surface to the surface of the liquid.

277. The pressure of water per square inch, is *.434 lb. Avoir. for each foot below the surface* and per square foot is *62.5 lbs. Avoir. for each foot below the surface.*

278. The average pressure upon the side of a vessel filled with water, is *the pressure at a point one-half the depth of the water.*

NOTE.—The term *Pressure* as used by mechanical engineers, means pressure per square inch.

ILLUSTRATIVE EXAMPLES.

What is the pressure at a faucet 96 ft. below the level of the water in a tank?

SOLUTION.

The pressure 1 ft. below the surface is *.434 lb.*; hence the pressure at 96 ft.
 $= 96 \times .434 \text{ lb.} = 41.664 \text{ lb.}$ Ans.

What is the total pressure on the bottom of a tank 8 ft. square, the water being 10 ft. deep in the tank?

SOLUTION.

$10 \times 62.5 \times 8 \times 8 = 40000 \text{ lbs.}$ Ans. EXPLANATION.—The pressure per square foot is $10 \times 62.5 \text{ lbs.}$; this multiplied by (8×8) , the area of the bottom, is 40000 lbs.

What is the total pressure upon one side of a tank 12 ft. square and 16 ft. high, when filled with water?

SOLUTION.

$8 \times 62.5 \text{ lbs.} \times 12 \times 16 = 9600 \text{ lbs.}$ Total pressure.

EXPLANATION.— $8 \times 62.5 \text{ lbs.}$ = average pressure per square foot on the side of the tank; this multiplied by 12×16 , the area of the side, gives the total pressure.

EXAMPLES FOR DRILL.

1. A box 8 ft. long, 6 ft. wide, and 5 ft. deep, is filled with water. Find the total pressure on the bottom and sides.

2. Find the total pressure exerted through a 4 in. fire-hydrant 124 ft. below the "head," or level of the water in the reservoir.

SUGGESTION.—Multiply the area of the cross-section of the pipe by the pressure per inch.

3. A milldam is 26 ft. high and 95 ft. long, what is the total pressure when the water is on a level with the top of the dam? What is the inch-pressure at the bottom of the dam?

4. A tube 4 ft. 6 in. high is filled with mercury. Find the inch-pressure at the bottom of the tube, mercury being 13.6 times as heavy as water.

LONGITUDE AND TIME.

279. Owing to the rotation of the earth, the sun appears to move around the earth in a circle, once in 24 hours.

The sun, therefore, appears to move through $\frac{1}{24}$ of 360° , or 15° in 1 hour. Hence, if it is noon at a certain place, 15° east of that place it will be one hour *after* noon, or 1 o'clock P. M.; 30° east of that place it will be two hours after noon, or 2 o'clock P. M. etc. Also, 15° west of that place it will be one hour before noon, or 11 o'clock A. M.; and 30° west, it will be two hours before noon, or 10 o'clock A. M. etc.

It follows, therefore, that at a given time of the day at any place, it will be *after* that time at all places east of that place, and *before* that time at all places west of that place.

280. The Meridian of a place is a line running north and south through that place.

281. The Longitude of a place is its distance, expressed in circular measurement units, east or west of some given meridian. For convenience it is customary to take the meridian passing through the Observatory at Greenwich, near London, Eng., as the meridian from which to reckon longitude. Places east of this meridian are said to be in East Longitude (Lon. E.), places west of this meridian are said to be in West Longitude (Lon. W.).

282. *The difference in longitude between two places is found by subtracting their longitudes, if they are on the same side of the Greenwich meridian, or by adding their longitudes if they are on opposite sides.*

283. **Local Time** is the time of a place reckoned from the rising of the sun at that place. Two places can have the same local time, only when they are situated upon the same meridian.

284. The difference in local time between places having different longitudes, may be easily calculated by either of the following—

TABLES OF LONGITUDE AND TIME.

TABLE I.

A difference in time of	A difference in longitude of
1 hour	= 15°
1 minute	= 15'
1 second	= 15"

TABLE II.

A difference in longitude of	A difference in time of
1°	= 4 minutes.
1'	= 4 seconds.
1"	= $\frac{1}{15}$ second.

285. To find Difference in Time when Longitude is given.

ILLUSTRATIVE EXAMPLE.

Dubuque, Ia., is in Lon. 90° 38' W., and San Francisco is 122° 23' W. Required the difference of their local time.

SOLUTION.

$(122^{\circ} 23') - (90^{\circ} 38') = 31^{\circ} 45'$ dif. in Lon.

$31^{\circ} 45' \div 15 = 2$ hr. 7 min. Ans.

EXPLANATION.—Subtracting the longitudes of the cities we get 31°

45', their difference in longitude. Dividing the units of longitude by 15 (Tab. I) we get the corresponding units of time, 2 hr. 7 min.

We can get the difference in time by Table II, thus:—

$$31^{\circ} \times 4 = 124 \text{ min.} = 2 \text{ hr. } 4 \text{ min.}$$

$$45' \times 4 = 180 \text{ sec.} = 3 \text{ min.}$$

$$\text{Diff. in time} = 2 \text{ hr. } 7 \text{ min.}$$

The first of these methods is usually preferred, although the second method is often convenient.

EXAMPLES FOR DRILL.

1. Portland, Maine, is in Lon. $70^{\circ} 15' W.$, Astoria, Oregon, is in $123^{\circ} 48' W.$ Find their difference in time.
2. If it were 2.40 P. M. at Portland, what would be the hour at Astoria?
3. If it were 7.15 P. M. at Astoria, what would be the time at Portland?
4. St. Petersburg is in Lon. $30^{\circ} 19' E.$, and Washington is in Lon. $77^{\circ} 15' W.$ When it is noon at Washington what is the time at St. Petersburg?
5. A sets his watch by the town clock in a certain city, and then journeys $51^{\circ} 40'$ west. If his watch keeps correct time will it be slower or faster than the local time of the place where he stops, and how much?

286. To find Difference in Longitude when Difference of Time is given

ILLUSTRATIVE EXAMPLE.

The difference in time between two places is 3 hr. 5 min. 6 sec.; what is their difference in longitude?

SOLUTION.

hr. min. sec.

$$(3 \quad 5 \quad 6) \times 15 = 46^{\circ} 16' 30'' \text{ Ans.}$$

EXPLANATION.— Multiplying the units of time by 15 (Table I) we get the corresponding units of longitude

EXAMPLES FOR DRILL.

1. The difference in time between two places is 1 hr. 17 min. 12 sec. Find their difference in longitude.
2. When it is 9.15 A. M. at A, it is 1.20 P. M. at B What is the difference in longitude between the two places?
3. In what direction and over how many units of longitude will I travel, in order that my watch may be 2 hr. 25 min. faster than the local time of the place where I stop?

STANDARD TIME.

287. When the different states and cities of the United States and Canada became intimately connected by railway systems, in order to avoid confusion, it became necessary to adopt a uniform or Standard Time in each of the several sections of the country.

For the Atlantic Seaboard and adjacent states Eastern Time, or the local time of the 75th meridian, was chosen. For the states of the Mississippi Valley, Central Time, or the local time of the 90th meridian; for the states and territories of the Rocky Mountain region Mountain Time, or the local time of the 105th meridian, and for the Pacific Coast, Pacific Time, or the local time of the 120th meridian, was adopted.

As these meridians are consecutively 15 degrees apart, it follows that the time throughout each "time belt" will be one hour later than the time of the belt west of it. Thus, when it is noon at New York, it will be 11 o'clock A. M. Standard Time, at all points in the Mississippi Valley, 10 o'clock in Rocky Mountain cities, and 9 o'clock A. M. throughout the Pacific Coast.

Local Time will agree with Standard Time, only at points situated upon the selected meridians, 75°, 90°, 105°, and 120°.

In many cities, both Local and Standard Time are used. They may vary as much as thirty minutes.

EXAMPLES FOR DRILL.

(All questions refer to standard time unless otherwise stated.)

1. When it is 10.45 A. M. at San Francisco what is the time at New York?
2. When it is 4.20 P. M. at Chicago, what is the time at Denver?
3. An eastern city is in Lon. 70°; how does its local time compare with its standard time?

SOLUTION.

Since the city is 75° — 70° or 5° east of the eastern standard meridian, its local time will be 5×4 , or 20 min. *faster* than the standard time of that meridian.

4. The longitude of Washington is 77° 15'. Is its standard time slower or faster than its local time, and how much?

5. Chicago is Lon. 87° 30'. When it is noon, standard time, what is the hour, local time?

288. GENERAL REVIEW.

TABLE OF EQUIVALENTS FOR PRACTICAL MEASUREMENTS.

1 even bu.	= $1\frac{1}{4}$ cu. ft.	1 pch. of stone	= $24\frac{1}{2}$ cu. ft.
1 cu. ft.	= $\frac{1}{8}$ bu.	1 square	= 100 sq. ft.
1 cu. ft.	= $7\frac{1}{2}$ gal.	1 cu. yd. of earth	= 1 load.
1 cu. ft.	= $\frac{5}{8}$ bbl.	1 cu. ft. of timber	= 12 bd. ft.
1 T settled hay	= 512 cu. ft.	28 bu. of coal	= 1 T.
1 T. loose hay	= 1000 cu. ft.	$2\frac{1}{2}$ cu. ft.	= 1 bu. corn in the ear.
1 cd. of wood	= 128 cu. ft.		

WEIGHT OF GRAINS, SEEDS, ETC., PER BUSHEL.

(The weights given are those used in most of the states.)

Articles.	Wt. per bu.	Articles.	Wt. per bu.	Articles.	Wt. per bu.
Barley,	48	Corn, in ear,	70	Onions,	57
Beans,	60	Drd. Apples,	28	Potatoes,	60
Blue Grass,	14	Drd. Peaches,	33	Rye,	56
Buckwheat,	40 to 52	Flax seed,	56	Salt,	50
Clover Seed,	60	Hemp,	44	Timothy seed,	45
Corn, shelled,	56	Oats,	32	Wheat,	60

PROBLEMS.

1. A nugget of gold weighs 1 lb. 9 oz. Avoir. What is its value at \$18 per Troy ounce?
2. A steamer left port at 8.25 A. M. Thursday and reached her destination at 3.15 P. M. the following Monday. What was the exact time of her voyage?
3. How many long tons of coal may be contained in a bin 12 ft. 6 in. long, 7 ft 8 in. wide, and 6 ft. high, the coal weighing 80 lbs. to the bushel?
4. At \$13.75 per T. find the value of a mow of hay 48 ft. \times 36 ft. and 13 ft. deep, estimating 500 cu. ft. to the ton.
5. A wagon-bed 10 ft. 8 in. long, 3 ft. 6 in. wide, and 21 in. deep is filled with wheat. What is the weight of the load?
6. At \$21.75 per ton, what is the value of potatoes in a bin 46 ft. \times 8 ft. 6 in. and 6 ft. deep?
7. Mr. A left a city in Lon. $123^{\circ} 20' W.$ at 8.35 A. M. local time, Jan. 16. He arrived at a city in $80^{\circ} 15' W.$ at 3.15 P. M. local time on Jan. 25. What was the exact time of his journey?

8. Find the capacity in barrels of a cistern $5\frac{1}{2}$ ft. in diameter and 21 ft. deep.

9. At \$12.50 per M., what cost the lumber for a 6 ft. side-walk around the outside of a lot 240×360 ft., the walk to be made of 2 in. plank and to rest on 2 stringers 3×6 in.?

10. A circular race-track is 1 mi. long. How many acres in the space enclosed by it?

11. A piece of land is 1567 ft. long and 849 ft. wide. What is it worth at \$75 per acre?

12. At \$3.75 per cord, what is the value of a pile of 4 ft. wood, 7 ft. high and 20 rds. long?

13. A vessel is in Lon. $21^{\circ} 50' 22''$ E. and sailed west $95^{\circ} 17' 40''$. What was her longitude then?

14. The following lots of land were sold for taxes at \$15.60 per acre. $n\frac{1}{2}$ s e $\frac{1}{4}$ Sec. 14; s w $\frac{1}{4}$ n e $\frac{1}{4}$ Sec. 11; s $\frac{1}{2}$ Sec. 6, s w $\frac{1}{4}$ Sec. 13; s $\frac{1}{2}$ n e $\frac{1}{4}$ s w $\frac{1}{4}$ Sec. 18. What sum was realized by the sale?

15. How many barrels of water fell on a roof 60×125 ft. during a three-inch rain?

16. On Monday morning, Apr. 29, 1895, Harvey Brown commenced working for George Wilson at \$60 per working-month. He boarded with his employer, paying \$3.50 per week. May 25, Brown was paid cash \$35, and on June 6, \$40. During the term of his employment Brown lost $4\frac{1}{2}$ days. What sum was due Brown on the evening of July 3, 1895?

PERCENTAGE.

289. Percentage embraces that large class of computations in which one quantity is considered as hundredths of another quantity.

290. The term **Per Cent** (%) is derived from the Latin phrase **per centum** (by the hundred) and means "hundredths." Per Cent rates are simply decimal fractions having 100 for the denominator, and may be written decimally, or as common fractions in lower terms. Thus, $25\% = \frac{25}{100}$, and may be written .25, or may be reduced to and considered as $\frac{1}{4}$.

291. Some per cent rates are best treated as common fractions, especially those that are aliquot parts of 100. Following is a table of these, which should be *thoroughly committed*.

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 20\%$	$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$
$\frac{1}{4} = 33\frac{1}{3}\%$	$\frac{2}{5} = 40\%$	$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{15} = 6\frac{2}{3}\%$
$\frac{1}{4} = 25\%$	$\frac{4}{5} = 80\%$	$\frac{1}{10} = 10\%$	$\frac{1}{20} = 5\%$
$\frac{3}{4} = 75\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{40} = 2\frac{1}{2}\%$

Other per cent rates are best expressed as decimals.

292. To express any common fraction as a per cent rate, *annex two ciphers to the numerator and divide the result by the denominator*.

Thus, $\frac{3}{8} = \frac{3.00}{8} = 37\frac{1}{2}\%$. Conversely, to change any per cent rate to a fraction, supply the denominator, 100, and reduce to lowest terms; thus, $28\% = \frac{28}{100} = \frac{7}{25}$.

293. To express any per cent rate as a **Decimal**, point off **two places**; thus, $3\% = .03$; $2\frac{1}{3}\% = .02\frac{1}{3}$; $6\% = .006$; $5.7\% = .057$, etc.

294. The **Terms** in percentage computations are three:—

I. **Base**, or the quantity to which the per cent rate applies.

II. **Rate**, or the number of hundredths expressed by the per cent rate.

III **Percentage**, or the product obtained by multiplying the Base by the Rate expressed as a fraction or a decimal.

ILLUSTRATION—In the expression 25% of \$12, \$12 is the *Base*, 25 is the *Rate*, and $\$12 \times .25$ (or $\frac{1}{4}$) = \$3, the *Percentage*.

NOTE.—The terms Base, Rate, and Percentage, correspond mathematically to the terms Base, Relation, and Part, used in "Relation of Numbers," p. 61.

295. Computations in Percentage depend upon the principles of "Relation of Numbers." (See Art. 131.)

These, as applied to Percentage, are:—

I. *The Base multiplied by the Rate equals the Percentage.*

II. *The Percentage divided by the Rate equals the Base.*

III. *The Percentage divided by the Base equals the Rate.*

296. Problems in Percentage, like those of "Relation of Numbers," may be solved either by logical analysis, or by formulas. The student should become familiar with both methods.

297. To find the Percentage when the Base and Rate are given.

ILLUSTRATIVE EXAMPLE.

What is 13% of \$400?

LOGICAL ANALYSIS.—1% of \$400 is \$4, and 13% is, therefore, 13 times \$4, or \$52. Ans.

FORMULA.—Base \times Rate = Percentage.
 $\$400 \times 13\% (= 13) = \52 Ans.

NOTE—If the rate is found in the table (Art. 291); use its fractional equivalent, and solve the problem in the same manner as in Relation of Numbers.

EXAMPLES FOR DRILL.

What is—

- | | | |
|-----------------------------------|------------------------------------|---------------------------------|
| 1. 8% of \$155.00? | 4. $66\frac{2}{3}\%$ of \$366.60? | 7. 50% of \$156.78? |
| 2. 9% of \$105.00? | 5. $87\frac{1}{2}\%$ of \$384.24? | 8. 38% of \$1400.00? |
| 3. $12\frac{1}{2}\%$ of \$348.80? | 6. $33\frac{1}{3}\%$ of \$1000.00? | 9. $6\frac{2}{3}\%$ of \$75.60? |

10. A man had \$1500, and invested $62\frac{1}{2}\%$ of it in bonds. Find amount invested in bonds.

11. A house cost \$3000, $37\frac{1}{2}\%$ of the cost was paid in cash, and a mortgage given for the remainder. What was the amount of the mortgage?

12. A farmer had 320 acres of land, and sowed 20% of it with wheat, and $33\frac{1}{3}\%$ of the remainder with oats. How many acres were there of each kind of grain, and how much remained?

13. A mechanic has an income of \$1000 a year; he pays 25% of it for board, $\frac{1}{2}\%$ for washing, gives $\frac{1}{10}\%$ to charity, pays 20% of it for clothes, and 10% for other expenses. What does each item cost, and what has he left?

14. John earns a salary of \$75, spends 12% for clothes, 25% for board, and $16\frac{2}{3}\%$ for other expenses. What sum does he save?

298. To find the Base when the Rate and Percentage are given.

ILLUSTRATIVE EXAMPLE.

\$16.40 is 8% of what sum?

LOGICAL ANALYSIS.—1% of the required sum = $(\$16.40 \div 8)$, or \$2.05, and 100%, or the required sum, is $\$2.05 \times 100$, or \$205. Ans.

FORMULA.—Percentage \div Rate = Base.
 $\$16.40 \div 8\% (= .08) = \$205.$ Ans.

EXAMPLES FOR DRILL.

- | | |
|--------------------------------|--|
| 1. \$82 is 10% of what number? | 4. \$13.25 is 25% of what number? |
| 2. \$75 is 15% of what number? | 5. \$34.65 is 35% of what number? |
| 3. \$83 is 20% of what number? | 6. \$136.80 is $37\frac{1}{2}\%$ of what number? |

7. A farmer sold 36 acres of land, which was 18% of the whole farm. How many acres were there in the farm?

8. I sold $37\frac{1}{2}\%$ of my interest in a factory for \$3,690. At the same rate, what was the value of my whole interest, and what is the value of my remaining interest?

9. A merchant invested 15% of his capital in a lot, 30% of it in erecting a building on the lot, 45% of it in a stock of groceries, and has \$3360 remaining. What was the total amount of his capital and the amount of each investment?

10. A clerk's salary is now \$540 a year, which is only $83\frac{1}{3}\%$ of what was formerly paid him. What was his former salary?

11. A coal mine was sold for \$56000, which was $87\frac{1}{2}\%$ of what it cost. What did the mine cost?

299. To find the Rate when the Base and Percentage are given.

ILLUSTRATIVE EXAMPLE.

\$12 is what per cent of \$32?

LOGICAL ANALYSIS.—1% of \$32 is 32¢; then \$12 is as many per cent of \$32 as 32¢ is contained times in \$12, or $37\frac{1}{2}\%$. Ans.

Also: \$12 is $\frac{12}{32}$, or $\frac{3}{8}$, of \$32. $\frac{3}{8}$ of 100% = $37\frac{1}{2}\%$. Ans.

FORMULA.—Percentage \div Base = Rate.
 $\$12 \div 32 (= .375) = 37.5\%$.

EXAMPLES FOR DRILL.

1. What per cent of \$560 is \$112? \$168? \$448?
2. What per cent of \$480 is \$357.60? \$171?
3. What per cent of \$450 is \$247.50?
4. A collector charged \$54.60 for collecting a bill of \$910. What per cent did he charge for collection?
5. The population of a city increased from 12,000 to 14,880. What was the rate per cent of increase?
6. A person's income is \$1240 a year, and his expenses are \$793.60. What per cent of his income does he save?
7. A merchant has \$25000 invested, and the income arising therefrom is \$3125. What rate per cent of increase does his investment produce?
8. A business man is worth \$36500, of which amount he has \$13687.50 invested in his business, and the remainder in real estate. What per cent of his capital is invested in real estate?

REVIEW EXAMPLES.

1. A merchant bought an invoice of goods amounting to \$2250, and paid \$30 freight on the goods. What per cent of the invoice price were the freight charges?
2. By assessing a tax of $\frac{1}{8}\%$, \$35490 were raised, what was the value of the property taxed?
3. A clerk's salary was raised 15%, and he now receives \$1495. What was his salary at first?
4. A grocer sold 15% of an invoice of coffee at one time, 35% at another, $37\frac{1}{2}\%$ at another, and had 85 lbs. of coffee remaining. How many pounds of coffee did he buy?
5. A and B enter into partnership; A invests \$6192 and B \$9283. What per cent of the capital did each invest?

6. A debtor owed \$5000 and agreed to pay the debt in equal instalments of $12\frac{1}{2}\%$ each. How much did he owe after paying seven instalments?

7. A farmer had 3250 sheep, and sold 1150. What per cent did he have remaining?

8. According to the census of 1880, a city had a population of 56160; in 1890 the population was 74880. What was the rate per cent of increase?

9. On Nov. 26, 1894, a merchant bought an invoice of goods amounting to \$958.40 on 3 months' credit, or 3% off, if paid within 30 days from date of sale. What amount would pay the bill Dec. 17, 1894?

10. A coal-dealer's sales for the month of November were 136400 pounds of coal; the sales for December were 196000 pounds. What was the rate per cent of increase?

11. An insurance agent retained \$25, which was $12\frac{1}{2}\%$ of the amount collected. How much did he collect?

12. I bought a lot for \$3600, and sold it for $37\frac{1}{2}\%$ more than it cost. What did I receive for it?

13. Two railroad companies carried 4928 lbs. of freight at the rate of 42¢ per cwt. If one company received $37\frac{1}{2}\%$ of the charges, what amount would the other company receive?

14. The charge for a certain kind of through freight, a distance of 500 miles, was 85¢ per cwt.; the freight was handled by three railroads and the freight charges were divided according to distance carried, which was 125, 175, and 200 miles respectively. What per cent of the total charges did each receive? If freight weighing 4560 lbs. was carried, what amount did each receive?

15. A creditor, after collecting $22\frac{1}{2}\%$ of a claim, lost the remainder, which was \$1550. What was the amount of the claim?

16. A ranchman had 3300 sheep killed during a storm, which was $1\frac{1}{2}\%$ of the entire flock. How many sheep were in the flock?

17. A manufacturer's sales amounted to \$163400 for the month of October; the sales for November showed an increase of 5% over the previous month, and the sales for December showed a decrease of 3% as compared with November sales. What was the rate per cent of increase of the December sales as compared with October?

18. On account of shrinkage in values, a capitalist's income is reduced \$1250, which is $12\frac{1}{2}\%$ of his former income. What was his former income?

19. The assets of a firm are \$35,137.50, and its liabilities \$42,165. What per cent of its indebtedness can the firm pay?

20. A farmer sowed 1 bu. 1 pk. of wheat to the acre and received 25 bu. to the acre. What was the rate per cent of increase?

SUGGESTION.—Here the Base is the number of bushels sowed.

COMMERCIAL, OR TRADE DISCOUNT.

300. Discount is a percentage allowance, or deduction made from the amount of a debt.

301. Trade Discounts are allowances made by wholesalers, jobbers, manufacturers, publishers and others, from the catalogue, or list price of goods.

NOTE.—These discounts are usually made from fixed list prices, in order to make the net prices conform to varying market quotations. Discounts are also made in accordance with varying terms of credit.

302. Sometimes several consecutive discounts are allowed at one time. These are known as **Series Discounts**. For instance, the terms on a bill may read: "20%, 10%, and 5% off." This means that a discount of 20% is allowed off the amount of the bill at the list price, then a further discount of 10%, and finally a still further discount of 5%. These discounts may be allowed as a special inducement for trade, or to adjust prices to a falling market, etc. They are not to be added and calculated as a single discount, but must be calculated consecutively as in the following example.

ILLUSTRATIVE EXAMPLE.

Find the net cost of an invoice of goods amounting to \$600, the buyer being allowed a series discount of 20%, 10%, and 5%, from list prices.

FIRST SOLUTION.

\$600 \times 80% = \$480, 1st net cost.

\$480 \times 90% = \$432, 2d net cost.

\$432 \times 95% = \$410.40, 3d net cost. Ans.

EXPLANATION.—The net cost at the first discount of 20% is 100% — 20% = 80% of \$600, or \$480; the net cost

at the second discount of 10% is 100% — 10%, or 90% of \$480 = \$432; the net cost at the third discount of 5% is 100% — 5%, or 95% of \$432 = \$410.40.

SECOND SOLUTION.

$\$600 \times 20\% = \120 , 1st dis.
 $\$600 - \$120 = \$480$.
 $\$480 \times 10\% = \48 , 2d dis.
 $\$480 - \$48 = \$432$.
 $\$432 \times 5\% = \21.60 , 3d dis.
 $\$432 - \$21.60 = \$410.40$. Ans.

EXPLANATION.—Here the amount of each discount is found separately and subtracted. This solution is usually preferred by accountants, although it is frequently longer than the first method

THIRD SOLUTION.

$80\% \times 90\% \times 95\% = 68.4\%$ net cost rate.
 $\$600 \times 68.4\% = \410.40 . Ans.

EXPLANATION.—After deducting the first discount of 20%, there

will remain 80% of the cost. Deducting 10%, the second rate leaves 90% of what remained after the first deduction. Deducting 5%, the third rate leaves 95% of what remained after the second deduction. The net cost will therefore be 95% of 90% of 80% = 68.4% of \$600 = \$410.40. This method of solution is preferred by many.

303. The Net Cost Rate is the per cent that the net cost is of the list, or gross cost. Thus, in the last example, 68.4% is the Net Cost Rate. **The Net Discount Rate** is the rate that the whole discount allowed is of the list cost. It is found by subtracting the net cost rate from 100. Thus, in the last example, $100\% - 68.4\% = 31.6\% =$ Net Discount Rate.

NOTE.—The results and terms of a discount series are the same, regardless of the order in which they are taken.

EXAMPLES FOR DRILL.

Find the net selling-price.

Gross price.	Discount off.	Gross price.	Discount off.
1. \$320,	25% and 10%	6. \$378.20,	10%, 5% and 3%.
2. \$420,	20% " 5%	7. \$492.50,	20%, 15% " 10%.
3. \$664 50,	10% " 5%	8. \$743.24,	40%, 10% " 10%.
4. \$982.50,	20% " 20%	9. \$928.30,	60%, 10% " 5%.
5. \$364.40,	15% " 10%	10. \$845.60,	50%, 30% " 10%.

11. An invoice of goods amounting to \$500 was sold on Dec. 18, 1894, at a discount of 20%, 10%, and 5%, with an additional discount

of 2% if paid within 60 days, or 5% if paid within 30 days from date of purchase. How much would pay the bill Jan. 15, 1895?

12. The wholesale dealer's price for cloth is \$3.50 per yard, subject to a discount of 20%, 10%, and 5%. What is the net price?

13. Find net cost of a bill of hardware invoiced at \$934.50, with discounts of 30%, 10%, and 10%.

14. If hats are listed at \$48 a dozen, subject to discounts of 40%, 10%, and 10%, what is the net selling-price?

15. A offers to sell lamps at \$10 a dozen, with discounts of 10%, 10%, and 5%; B offers the same grade of lamps for \$12 a dozen, with discounts of 20% and 20%. Which is the better offer, and how much would be saved on an invoice of 50 doz. lamps?

16. If goods which cost \$240 are marked \$300, what rate of discount must be allowed to net cost?

17. A bill of goods amounting to \$147 was sold on the following terms: 60 days, 5%; 30 days, 10%; cash, 20%. Find the net cash cost.

NOTE.—Discounts often vary according to the term of credit. These are sometimes known as "time discounts."

18. Find the cost of the bill in Example 17, if paid for at the end of 25 days.

19. The Ballin Hardware Co. sold R. Smith the following invoice of goods. Terms, 20% off list. 60 days, 5%; 30 days, 10%; cash, 15%.

5½ doz. Axes	\$7.60.
6¾ " Garden Rakes	\$4.20.
9½ " Grind Stones	\$.95.
2½ " Diston Saws	\$22.70.

What is the net cost of the above bill if paid at the end of 90 days? 45 days? 20 days? If paid on receipt of goods?

20. The net cost of a bill of goods was \$129.60 with a discount of 10% and 8% off. What was the list cost?

21. Buying goods at a discount of 20% and 10% off and selling them at list cost, I gained \$175. What was the list cost of the goods?

22. Bought a bill of goods amounting to \$267.50, receiving a discount of 20%, 15%, and 10% off. If I sold the goods for \$275, allowing my customer a discount of 20% and 10% off, what sum did I gain?

PROFIT AND LOSS.

304. **Losses and Gains** in business are usually expressed as percentages of the gross cost of goods. Thus to gain 8% upon a stock of goods is to sell them for 8% more, or 108% of their cost.

305. **Gross Profit, or Gross Gain**, is the difference between the cost and the sum received for an article.

306. **Net Profit, or Net Gain**, is the Gross Gain less all expenses.

307. **Net Cost** is the amount paid for the goods.

308. **Gross Cost** is the *Net Cost plus all expenses* incident to the purchase, as commission, duties, freight, drayage, etc.

309. **Gross Sales** is the total sum for which the goods are sold to the buyer and correspond to the buyer's *Net Cost*.

310. **Net Sales** is the Gross Sales less all expenses incident to the sale, as commission, drayage, etc., that may be paid by the seller.

311. **Price** is the value agreed upon by the buyer and seller for a single article, or for 1 lb., 1 ton, 1 bu., 1 yd., 1 doz. etc.

312. **Gross, or List Price** is the price on the seller's list, or catalogue.

313. **Net Price** is the List Price less any commissions allowed by the seller.

NOTE.—The student should make himself thoroughly familiar with the meanings of the above terms

314. **The Base** in Profit and Loss computations is in most general transactions the *gross cost*; though in marking goods, and in some other cases, it may be either the *net* or the *list* price. The **Percentage** is the *Net Gain* or *Net Loss*.

GENERAL RULES.

I. $\text{Cost} \times \text{Rate of Gain or Loss} = \text{Gain or Loss}.$

II. $\text{Gain or Loss} \div \text{Cost} = \text{Rate of Gain or Loss}.$

III. $\text{Gain or Loss} \div \text{Rate of Gain or Loss} = \text{Cost}.$

ILLUSTRATIVE EXAMPLES.

Bought goods for \$200 and sold them at a gain of 25%. Find gain and sales.

MODEL SOLUTION.

$$\$200 \times \frac{1}{4} (= 25\%) = \$50. \text{ Gain.}$$

$$\$200 + \$50 = \$250. \text{ Sales.}$$

Bought a horse for \$60 and sold it for \$70. What per cent did I gain?

MODEL SOLUTION.

$$\$70 - \$60 = \$10. \text{ Gain.}$$

$$\$10 \div \$60 = \frac{10}{60} = \frac{1}{6} = 16\frac{2}{3}\%. \text{ Ans.}$$

Sold a stock of goods at a net gain of 20%, and cleared \$175. Find cost and sales.

MODEL SOLUTION.

$$\$175 \div \frac{1}{5} (= 20\%) = \$875. \text{ Cost.}$$

$$\$875 + \$175 = \$1050. \text{ Sales.}$$

Sold goods for \$540 and gained 25%. What did they cost?

MODEL SOLUTION.

$$\$100\% + 25\% = 125\% = \text{per cent of Sales.}$$

$$\$540 \div 125\% = \left(\frac{5}{4}\right) = \$432. \text{ Cost.}$$

Sold goods for \$1700 and lost 15%. Find cost and loss.

MODEL SOLUTION.

$$100\% - 15\% = 85\% \text{ of Sales.}$$

$$\$1700 \div 85\% = \$2000 \text{ Cost.}$$

$$\$2000 - \$1700 = \$300. \text{ Loss.}$$

EXAMPLES FOR DRILL.

NOTE.—Let the student be on the watch for “short cuts” and opportunities for getting results mentally. Much labor may be saved by selecting the best method of solution.

1. Goods costing \$350 were sold at a gain of 12%. Find gain.
2. What is the cost of goods sold for \$1029 at a gain of 5%?
3. Goods costing \$180.00 were sold at a gain of 25%. Find gain.
4. Goods costing \$385.50 were sold at a gain of $33\frac{1}{3}\%$. Find gain.
5. Goods costing \$948.75 were sold at a gain of 40%. Find gain.
6. Goods costing \$740.00 were sold at a loss of $12\frac{1}{2}\%$. Find loss.
7. Goods costing \$366.60 were sold at a loss of $16\frac{2}{3}\%$. Find loss.
8. Goods costing \$348.40 were sold at a loss of 3%. Find loss.
9. Goods costing \$17.20 were sold at a loss of $6\frac{1}{4}\%$. Find loss.
10. I sold a farm for \$4572.50, making 25%, what did the farm cost?
11. If I pay \$125 for a horse, and sell it gaining 15%, what is my profit?
12. Bought a farm for \$9380 and sold it for \$11725. Find the gain per cent.
13. Find the cost of a watch which was sold for \$87.50, at a profit of $16\frac{2}{3}\%$.
14. What is the per cent of profit on a lot which cost \$700, and was sold for \$840?
15. A merchant sold a bill of goods for \$92.75 thus losing 6%. How much did he lose?
16. Goods which cost \$3650 are sold at a profit of \$730. What is the rate per cent of profit?
17. A house and lot cost \$3250 and were sold for \$3412.50. What was the per cent of profit?
18. Bought goods at 20% and 10% off and sold at list cost price. What per cent do I gain?
19. A carriage was sold for \$302.50, at a gain of $37\frac{1}{2}\%$. What was the cost of the carriage?

20. If $12\frac{1}{2}\%$ profit is made by selling a bicycle for \$135, what was the cost of the bicycle?

21. Potatoes costing 45¢ a bushel and selling at 54¢ a bushel, produce what per cent of profit?

22. If by selling cloth at \$2 a yard I make 25%, what was the cost of the cloth per yard?

23. Paper bought at \$1.85 a ream and retailed at 6 sheets for 5¢ will produce what per cent of profit?

24. A coal-dealer sold a quantity of coal for \$4982.40 and lost 40%. What did he pay for the coal?

25. A horse was sold for \$22 less than it cost, thereby causing a loss of 5%. What was the cost of the horse?

26. A capitalist, on an investment of \$15000, received in annual returns \$1875. What is his rate per cent of increase?

27. A grain-dealer bought 1245 bu. of wheat at 50¢ a bushel, and sold the entire lot for \$647.40. What was his per cent of gain?

28. A house was bought for \$2500, \$300 was paid for repairs, and then was sold at a profit of $12\frac{1}{2}\%$ on gross cost. Find amount gained.

29. A stock of goods costing \$12600, was sold at a profit of $16\frac{2}{3}\%$. If 8% of the sales was not collectable, what was the net gain or loss?

30. What price should be asked for coffee which cost 20¢ a pound, that a gain of 10% may be realized, after allowing 10% loss for roasting?

31. The U. S. Furniture Co. sold two parlor sets for \$400 each, losing 20% on one and gaining 20% on the other. Find the cost of each.

32. Bought a pair of horses and sold one of them for \$137.50 gaining 10%; and the other for \$128.80 losing 8%. What did the pair cost?

33. A grain-buyer sold a quantity of wheat at 15% profit, thereby making \$532.50. What was the cost of the wheat and what was the selling price?

34. At what price must I mark goods that cost 80¢ a yard in order that I can give a customer a discount of 20% and still gain 25% myself?

35. A real estate agent sold three houses at \$1575 each, at a profit respectively of 6%, 8%, and 12%. What was the total amount paid for the three houses?

36. A coal-dealer bought 25 tons of coal at \$2.50 per ton; the freight charges were \$1.75 per ton. If he sells the coal at \$6 per ton, what would be the rate per cent of profit on gross cost?

37. The owner of a house was offered \$2500 for his house, if accepted, the loss would have been $16\frac{2}{3}\%$; he afterward sold the house for \$3500. Find the rate per cent of loss or gain.

38. Bought goods at 25% and 20% off; sold at an advance of 30% on net cost and gained \$75. What sum would I have gained had I sold at 10% advance on the list cost?

39. An invoice of goods amounted to \$850; the freight charges on the goods were \$15, and \$5 was paid for delivery. If the goods are sold at an advance of 15% on net cost, what is the profit?

40. If pens cost 55¢ a gross and are retailed at 25¢ a quarter gross, what is the rate per cent of gain? If the pens were sold at 1¢ each, what would be the rate per cent of profit?

41. A contractor bought 64000 brick at \$12.50 per M., and sold $\frac{1}{2}$ of them at a gain of 35%, $\frac{1}{4}$ at a loss of 5%, and received \$175 for the remainder. Find the amount of his gain and total selling-price.

42. If bread is bought at 30¢ per dozen loaves, and is sold at 4¢ per loaf, what is the rate per cent of gain? If the bread had been sold at 6 loaves for 10¢ what would have been the rate per cent of loss?

43. A grocer bought 750 lbs. of coffee at 25¢ a pound, of which 10% was damaged, and sold at a loss of 10%; the remainder was sold at a gain of 10%. Find the net gain, and the amount received for the coffee.

44. A merchant bought a bill of goods and paid 30% of the amount in cash, and was allowed 90 days in which to pay the balance of \$777. If the goods were sold at an average profit of $16\frac{2}{3}\%$, what did he receive for the goods?

45. If a retail dealer sells an article for \$98.00, gaining 25%; and the wholesale merchant by selling to the retailer makes 12%, and the manufacturer sold to the wholesaler at $16\frac{2}{3}\%$ above its first cost; what was its first cost?

46. Three horses were bought for \$120, \$150, and \$184 respectively. If the first was sold at a gain of 18%, the second at a gain of 22%, and the third at a loss of $12\frac{1}{2}\%$, what was the net gain, and the total amount received for the horses?

47. A real estate agent bought a lot at 10% below its assessed value and sold it at an advance of 30% above the assessed value, thereby gaining \$405. What was the assessed value of the lot and what per cent did the dealer make on his investment?

48. Bought an invoice of 29 stoves at \$24 each; received a discount of 8% and 10% off, and sold at a gain of 20% on net cost. For what sum did I sell them? Had I sold each stove at 10% below list cost, what sum would I have gained?

49. Sold two horses for \$450 each; on one I gained 20% of its cost, and on the other I lost 20% of its cost. Did I lose or gain on both, and how much?

INTEREST.

315. Interest is a sum charged for the use of money.

316. The Principal is the sum upon which interest is computed.

317. The Amount is the sum of the principal and interest.

318. The Time is the period during which the principal bears interest.

319. The Rate of interest is the rate per cent that is charged for the use of the principal for a stated time, usually one year.

320. The Legal Rate is the rate permitted by law when by agreement the parties consent to pay interest but mention no rate. This rate is also collectable on all debts, whether by written agreement or otherwise, remaining unpaid after maturity unless another rate is agreed upon.

321. The Contract Rate is the highest rate allowed by the law of the state, which may be agreed upon in a contract.

322. Usury is the collection of a higher rate of interest than is allowed by law.

SIMPLE INTEREST.

323. Simple Interest is the interest on the principal alone for the whole time during which the interest is computed.

324. Ordinary Interest is computed on a basis of 360 days for a year, and is the legal basis of computation where there is no special contract to the contrary.

325. There are many methods employed by accountants for computing interest, no one of which is best for all cases. We illustrate several of the best of these, and advise the student to become familiar with each, as much time is often saved by applying the most direct method of solution. When the time, or a part of it, is expressed in days, observe the aliquot part of a month, as 5 days = $\frac{1}{6}$, 6 days = $\frac{1}{5}$, 10 days = $\frac{1}{3}$, etc.

ANALYTICAL METHOD.

326.

ILLUSTRATIVE EXAMPLES.

1. Find the interest on \$720 for 3 yrs. 4 mo. 15 da. at 10%.

MODEL SOLUTION.

$$\$720 \times \frac{1}{10} (= 10\%) = \$72 \text{ interest for 1 year.}$$

$$72 \div 12 = \$6 \text{ interest for 1 month.}$$

$$3 \text{ yr. 4 mo. 15 da.} = 40.5 \text{ mo.}$$

$$\$6 \times 40.5 = \$243.00 \text{ interest for the required time.}$$

2. Find the interest on \$1350 for 2 mo. 19 da. at 8%.

MODEL SOLUTION.

$$\$1350 \times 8\% = \$108 \text{ interest for 1 year.}$$

$$\$108 \div 360 = 30c. \text{ interest for 1 day.}$$

$$2 \text{ mo. 19 da.} = 79 \text{ days.}$$

$$30c. \times 79 = \$23.70 \text{ interest for the required time.}$$

CANCELATION METHOD.

327.

ILLUSTRATIVE EXAMPLES.

1. Find the the interest on \$480 for 1 yr. 2 mo. 12 da., at 10%.

SOLUTION.

$$\frac{\$480 \times 14.4 \times 10}{100 \times 12} = \$57.60.$$

EXPLANATION.—Write the principal, \$480, in the dividend. Put 12×100 beneath in the divisor, to get 1 month's interest at 1%; then in the dividend write

the number of months (14.4), and the rate, 10. This is simply the different steps of the Analytical Method expressed in the form of a Compound Division, or Cancellation.

NOTE.—It is more convenient to arrange the work vertically, thus:—

(Divisor.)	(Dividend.)
100	\$480
12	14.4
	10

The following arrangement of the terms will give the simple interest for any computation:—

(Divisor.)	(Dividend.)
100	Principal.
12 (or 360)	Time (mo., or da.)
	Rate.

2. Find the interest on \$720 for 6 mo. 10 da., at 10%.

MODEL SOLUTION.

$$\begin{array}{r|l} 100 & 720 \ 2 \\ 360 & 190 \\ & 10 \\ \hline & \$38 \text{ int. Ans.} \end{array}$$

EXPLANATION.—Here write 360 in the divisor, as the time 6 mo. 10 da. is best expressed in days (190 da.). Canceling, we have $2 \times 19 = \$38$. Int.

328. From an inspection of the arrangement of terms in the last example it is evident that the interest for any rate or time may always be obtained from one or the other of the following

FORMULAS.

I. $(\text{Prin.} \times \text{Time [in months]} \times \text{Rate}) \div 1200 = \text{Int.}$

II. $(\text{Prin.} \times \text{Time [in days]} \times \text{Rate}) \div 36000 = \text{Int.}$

NOTE.—When the number of days is divisible by 3, express the time in tenths of a month. Thus, 3 da. = .1 mo.; 6 da. = .2 mo.; 9 da. = .3 mo.; 21 da. = .7 mo., etc.

TWELVE PER CENT METHOD.

329. From the following table a variety of methods have been derived for the computation of interest:—

TABLE.

The interest on \$1 at 12% for 1 yr. = 12¢.
The interest on \$1 at 12% for 1 mo. = 1¢.
The interest on \$1 at 12% for 1 da. = $\frac{1}{3}$ m.

ILLUSTRATIVE EXAMPLE.

Find the interest on \$500 at 8% for 2 yr. 9 mo.

SOLUTION.

$$16\text{¢} + 6\text{¢} = 22\text{¢} \text{ interest on \$1 for time at 8\%}.$$

$$22 \times 500 = \$110 \text{ required interest.}$$

at 12% for 2 years is 24¢, and at 8% $\frac{2}{3}$ of 24¢, or 16¢. Also that the interest for 9 months at 8%, will be $\frac{3}{4}$ of 9¢, or 6¢; or the interest on \$1 for the whole time will be 22¢, and for \$500, $500 \times 22\text{¢}$, or \$110.

EXPLANATION.—From the table it is evident that the interest on \$1

330. The interest at 12% on any sum may also be obtained by the following

FORMULAS.

I. $\frac{1}{100}$ of the Principal \times the number of months = Interest.

II. $\frac{1}{1000}$ of the Principal \times $\frac{1}{3}$ the number of days = Interest.

NOTE.—To find the interest at any other rate, first find interest at 12% by the formula, and then take such a part of this as the given rate is of 12%. Thus: If the rate is 4%, take $\frac{1}{3}$ of the interest at 12%; if 9%, take $\frac{3}{4}$ of the interest at 12%, etc.

ILLUSTRATIVE EXAMPLES.

1. Find the interest on \$400, at 9%, for 1 yr. 9 mo.

SOLUTION.

$$\$4 \times 21 = \$84 \text{ int. at 12\%}.$$

$$\$84 \times \frac{3}{4} = \$63 \text{ req. interest.}$$

EXPLANATION.—According to Formula 1: $\frac{1}{100}$ of \$400 = \$4. $21 \times \$4 = \84 interest at 12%. $9\% = \frac{3}{4}$ of 12%. $\$84 \times \frac{3}{4} = \63 . Ans.

2. Find the interest at 10% on \$600 for 81 days.

SOLUTION.

$$60\text{¢} \times 27 = \$16.20 \text{ int. at 12\%}.$$

$$\$16.20 \times \frac{5}{6} = \$13.50 \text{ req. int.}$$

of 12%, hence the required interest is $\frac{5}{6}$ of \$16.20, or \$13.50.

EXPLANATION.—Applying Formula 2: $\frac{1}{1000}$ of \$600 = 60¢. $\frac{1}{3}$ of 81 da. = 27 da.; $27 \times 60\text{¢} = \$16.20$. $10\% = \frac{5}{6}$

SIX PER CENT METHOD.

331. This method in some of its forms is used exclusively by many accountants.

TABLE.

The interest on \$1 for one year at 6% is \$.06.

The interest on \$1 for one month is \$.005, or $\frac{1}{2}\text{¢}$.

The interest on \$1 for one day is \$.000 $\frac{1}{6}$, or $\frac{1}{6}$ of a mill.

ILLUSTRATIVE EXAMPLE.

Find the interest on \$600 for 2 years, 6 months and 18 days at 6%.

SOLUTION.

Int. on \$1 for 2 yr. = \$.12.

Int. on \$1 for 6 mo. = \$.03.

Int. on \$1 for 18 da. = \$.003.

Int. on \$1 for 2 yr. 6 mo. 18 da. = \$.153.

Int. on \$600 for 2 yr. 6 mo. 18 da. = $600 \times \$.153 = \91.80 .

EXPLANATION.—Since the interest on \$1 for 1 year is \$.06, for 2 years it will be \$.12; since the interest on \$1 for 1 month is $\frac{1}{2}\text{¢}$, for 6 months it will be \$.03; since the interest for 1 day is \$.000 $\frac{1}{6}$, for 18 days it will be \$.003; therefore the interest on \$1 at 6% for 2 yr. 6 mo. 18 da. is \$.153, and the interest on \$600 will be 600 times the interest on \$1, or $600 \times \$.153 = \91.80 .

332. Interest at 6% may also be computed by the following

FORMULAS.

I. $\frac{1}{100}$ of the Principal $\times \frac{1}{2}$ the number of months = Interest.

II. $\frac{1}{1000}$ of the Principal $\times \frac{1}{6}$ the number of days = Interest.

NOTE.—To find interest at other rates, multiply the result at 6% by the relation that the given rate bears to 6. Thus, for 8% multiply by $\frac{4}{3}$, or add $\frac{1}{3}$ to the result at 6%; for 10%, add $\frac{1}{3}$ to the result, or multiply the result by $\frac{5}{3}$, etc.

ILLUSTRATIVE EXAMPLES.

1. Find the interest on \$960 for 1 yr. 6 mo. at 10%.

SOLUTION.

$\$9.60 \times 9 = \86.40 int. at 6%.

$\$86.40 \times \frac{5}{3} = \144 . Ans.

$\$86.40 \times \frac{5}{3} = \144 required interest.

EXPLANATION.— $\frac{1}{100}$ of \$960 = \$9.60;
 $\frac{1}{2}$ of 18 mo. = 9 mo. $\$9.60 \times 9 = \86.40
 int. at 6%. 10% = $\frac{5}{3}$ of 6%; hence

2. Find the interest on \$400 for 42 days at 8%.

SOLUTION.

$$40¢ \times 7 = \$2.80 \text{ int. at } 6\%.$$

$$\$2.80 \times \frac{4}{3} = \$3.73. \text{ Ans.}$$

EXPLANATION.— $\frac{1}{1000}$ of \$400 = 40¢. $\frac{1}{3}$ of 42 days = 7 days. $40¢ \times 7 = \$2.80$ int. at 6%. $8\% = \frac{4}{3}$ of 6%. $\$2.80 \times \frac{4}{3} = \3.73 req. int.

EXAMPLES FOR DRILL.

Find, by both the Analytical and Cancellation Methods, the interest on—

- | | |
|-----------------------------------|----------------------------------|
| 1. \$500, 1 yr. 2 mo. 12 da. 8%. | 4. \$840, 3 yr. 1 mo. 9 da. 9%. |
| 2. \$720, 2 yr. 5 mo. 18 da. 10%. | 5. \$960, 1 yr. 7 mo. 6 da. 10%. |
| 3. \$175, 4 yr. 2 mo. 7%. | 6. \$57.25, 28 da. 6%. |

Find, by both forms of the Twelve Per Cent Method, the interest on—

- | | |
|----------------------------------|-----------------------------------|
| 7. \$750, 1 yr. 6 mo. 6 da. 8%. | 10. \$24.60, 3 yr. 27 da. 12%. |
| 8. \$950, 2 yr. 7 mo. 10%. | 11. \$128, 2 yr. 5 mo. 15 da. 9%. |
| 9. \$850, 1 yr. 2 mo. 10 da. 6%. | 12. \$96.50, 129 da. 8%. |

Find, by both forms of the Six Per Cent Method, the interest on—

- | | |
|------------------------------|---|
| 13. \$750, 1 yr. 9 mo. 6%. | 16. \$680, 3 yr. 7 mo. 14 da. 9%. |
| 14. \$960, 63 da. 8%. | 17. \$375, 1 yr. 7 mo. $4\frac{1}{2}\%$. |
| 15. \$245, 5 mo. 27 da. 10%. | 18. \$240.20, 1 yr. 7 mo. 5%. |

333. In order to secure accuracy in computing interest, it is best to solve each example by two different methods, unless the computation is so simple as not to require this precaution. Solve the following, testing your results in each example, by means of a solution by another method.

EXAMPLES FOR DRILL.

Find the interest on—

- \$300 for 1 yr. 6 mo. 12 da. at 6%.
- \$350 for 1 yr. 4 mo. 18 da. at 6%.
- \$756.50 for 3 yr. 4 mo. 24 da. at 6%.
- \$954.60 for 6 yr. 6 mo. 6 da. at 6%.
- \$1000 for 3 yr. 7 mo. at 6%.

6. \$540.80 for 9 mo. 15 da. at 6%.
7. 346.50 for 7 mo. 19 da. at 6%.
8. \$464.75 for 3 yr. 9 mo. 23 da. at 6%.

Finding the time by compound subtraction, compute the interest at 6% on—

9. \$365.40 from Sept. 1, 1891 to Nov. 15, 1894.
10. \$420 from Mar. 3, 1889, to Sept. 14, 1892.
11. \$656.40 from Apr. 16, 1886, to Jan. 16, 1889.
12. \$325.75 from Jan. 3, 1889 to June 28, 1892.
13. \$386.50 from May 13, 1890 to Oct. 25, 1893.
14. \$964.40 from Nov. 18, 1891 to Feb. 6, 1894.
15. \$5216.80 from Dec. 12, 1892 to March. 15, 1895.

Finding the exact time in days, compute the interest at 6% on—

16. \$256.40 from Sept. 1, 1891 to Nov. 14, 1891.
17. \$389.40 from July 15, 1892 to Oct. 15, 1892.
18. Find the interest on \$1200 for 3 yr. 4 mo. 8 da. at $4\frac{1}{2}\%$.
19. What is the interest on \$525.25 for 4 yr. 3 mo. 8 da. at 8%?
20. Find the interest on \$2400 for 3 yr. 7 mo. at $7\frac{1}{2}\%$.
21. Compute the interest on \$14375 for 1 yr. 1 mo. at 5%.
22. Find the interest on \$364.25 for 11 mo. 10 da. at 10%.
23. What is the interest on \$375 from June 3, 1892 to Mar. 15, 1895, at 8%? (Find the time by compound subtraction.)
24. On the 15th day of Aug. 1892 I borrowed \$2500 at 7% interest. Find the total amount due Aug. 15, 1894.
25. What will \$1000 amount to in 2 yr. 6 mo. 9 da. with interest at 6%?

SHORT METHODS.

334. Many interest computations may be made mentally, and many others by a single multiplication or division, if one will take proper advantage of the Twelve Per Cent or the Six Per Cent Methods. To illustrate: What is the interest for 18 days on \$500 at 6%? Since the interest on \$1 at 6% is *one mill for each six days*, for 18 days it is 3 mills, and for \$500 is 500×3 mills, or \$1.50. Solve the following examples, taking advantage of this, and similar "short cuts."

EXAMPLES FOR DRILL.

Find the interest on—

- | | |
|------------------------------|--|
| 1. \$1450 for 60 days at 6%. | 9. \$390 for 2 days at 6%. |
| 2. \$394 for 30 days at 6%. | 10. \$400 for 3 days at 8%. |
| 3. \$376 for 30 days at 6%. | 11. \$1000 for 30 days at 9%. |
| 4. \$450 for 30 days at 6%. | 12. \$3600 for 60 days at $7\frac{1}{2}\%$. |
| 5. \$350 for 90 days at 6%. | 13. \$390 for 18 days at 6%. |
| 6. \$350 for 90 days at 7%. | 14. \$420 for 5 days at 6%. |
| 7. \$450 for 10 days at 6%. | 15. \$3900 for 9 days at 6%. |
| 8. \$670 for 6 days at 6%. | 16. \$4500 for 9 days at 8%. |

REMARK — In the following examples compute the interest to the nearest cent on each principal.

17. Find the total amount of interest on—

- | | |
|-----------------------------|-----------------------------|
| \$320 for 24 days at 6% | \$356 for 63 days at 6%. |
| \$450 for 30 days at 8%. | \$325 for 15 days at 4%. |
| \$1000 for 27 days at 5%. | \$352.40 for 33 days at 6%. |
| \$341.50 for 30 days at 9%. | \$324 for 90 days at 8%. |
| \$924.80 for 6 days at 8%. | \$500 for 36 days at 9%. |

18. Find the total amount of interest on—

- | | |
|---|--------------------------|
| \$1000 for 3 days at $4\frac{1}{2}\%$. | \$340 for 9 days at 6%. |
| \$900 for 57 days at 6%. | \$600 for 10 days at 8%. |
| \$366 for 2 mo. at 7%. | \$324 for 63 days at 8%. |
| \$960 for 1 mo. at 8%. | \$225 for 42 days at 6%. |
| \$400 for 31 days at 8%. | \$325 for 20 days at 8%. |

19. Find the total amount of interest on—

- | | |
|------------------------------|--|
| \$1500 for 2 mo. at 8%. | \$964 for 1 mo. at 6%. |
| \$3248.25 for 18 days at 7%. | \$456.90 for 15 days at $4\frac{1}{2}\%$. |
| \$1600 for 26 days at 7%. | \$1800 for 30 days at 6%. |
| \$2250 for 29 days at 8%. | \$5400 for 63 days at 6%. |
| \$2700 for 33 days at 8%. | \$320 for 54 days at 3%. |

20. Find the total amount of interest on—

\$3000 for 3 days at 8%.	\$5000 for 4 days at 6%.
\$9250 for 24 days at 5%.	\$7500 for 1 day at 6%.
\$12000 for 5 days at 5%.	\$256.40 for 48 days at 6%.
\$2000 for 36 days at 4%.	\$365 for 42 days at 6%.
\$100 for 27 days at 8%.	\$1000 for 35 days at 6%.

PROBLEMS IN INTEREST.

335. In making interest computations, there are four general quantities, or elements considered:—

I. Principal. II. Time. III. Rate. IV. Interest.

336. The relations of these quantities are such that if any three of them are given, the fourth may be found, and from these relations arise the

FOUR PROBLEMS IN INTEREST.

- I. *Principal, Time, and Rate given, to find Interest.*
- II. *Interest, Time, and Rate given, to find Principal.*
- III. *Interest, Principal, and Rate given, to find Time.*
- IV. *Interest, Principal, and Time given, to find Rate.*

337. The first of these problems is most important and has already been fully considered; but cases involving each of the other problems frequently arise.

PROBLEM II.

338. **Interest, Time, and Rate given, to find Principal.**

ILLUSTRATIVE EXAMPLE.

What sum, loaned for 2 yr. 8 mo., at 6% will produce \$24 interest?

SOLUTION.

Interest on \$1 for 2 yr. 8 mo. = \$.16

\$24 ÷ .16 = \$150 required principal.

SOLUTION BY CANCELATION.

(Dividend.) (Divisor.)

50 100	?
3 12	32 \$
4 \$24	6

$$50 \times 3 = \$150. \text{ Ans.}$$

product by the given factors in the other one. Hence arrange the terms according to the general form,

100	Principal,	and leave blank the space where the
12 (or 360)	Time in months (or days),	required term belongs. The terms
Interest,	Rate,	on that side of the vertical line on

which the missing term occurs, of course, constitute the *divisor*, and the terms on the other side are the *dividend*. This method applies to all interest problems, and obviates the necessity of either rules or analysis.

EXAMPLES FOR DRILL.

What principal will yield—

1. \$100 interest in 3 yr. 4 mo. at 6%?
2. \$6.08 interest in 60 da. at 8%?
3. \$72.80 interest in 120 da. at 5%?
4. \$65.70 interest in 1 yr. 4 mo. at 9%?
5. \$73 interest in 1 yr. 2 mo. 18 da. at 6%?
6. \$20.75 interest in 1 yr. 4 mo. 18 da. at 3%?
7. I borrowed a sum of money for 11 mo. 9 da. at 6% and paid \$8.24 interest. What was the amount borrowed?
8. How much money must be put at interest at 8%, Jan. 1, 1896, so that on Mar. 25, 1899, \$198.85 interest will be due?

PROBLEM III.

339. Interest, Principal, and Rate given, to find Time.

ILLUSTRATIVE EXAMPLE.

In what time will \$250 produce \$20 interest at 6%?

SOLUTION.

\$15 = int. on \$250 for 1 yr. at 6%.

\$20 ÷ 15 = 1⅓ yrs. = 1 yr. and 4 mo.

to produce \$20 as \$15 is contained times in \$20, or 1⅓ yrs. = 1 yr. and 4 mo. Or, the interest on \$250 for 1 da. at 6% is \$.04⅓; then it will require as many days to produce \$20 interest as 20 ÷ .04⅓ = 480 da. = 1 yr. 4 mo,

EXPLANATION.—We have seen, Art. 328, that (Prin. × Time [mo.] × Rate) ÷ 1200 = Interest. It is therefore evident that (Prin. × Time [mo.] × Rate) = (1200 × Int.), and that any missing factor in either of these compound products may be found by dividing the complete compound prod-

uct by the given factors in the other one. Hence arrange the terms according to the general form,

and leave blank the space where the required term belongs. The terms on that side of the vertical line on which the missing term occurs, of course, constitute the *divisor*, and the terms on the other side are the *dividend*. This method applies to all interest problems, and obviates the necessity of either rules or analysis.

CANCELATION.

$$\begin{array}{r|l} 4 & 100 & 250 \\ 4 & 12 & ? \\ & 20 & \$ 2 \end{array}$$

$4 \times 4 = 16 \text{ mo.} = 1 \text{ yr. } 4 \text{ mo.}$

EXPLANATION.—Leave the space for the time blank; then $(100 \times 12 \times 20) \div (250 \times 6) = 16 \text{ (mo.)} = 1 \text{ yr. } 4 \text{ mo.}$ Should the time be required in days, write 360 instead of 12 on the left hand side of the line.

EXAMPLES FOR DRILL.

1. In what time will \$350 produce \$70 interest at 6%?
2. How long will it take \$3600 to gain \$144 at 8%?
3. In what time will \$980 gain \$33.81 at 9%?
4. In what time will \$950 loaned at $4\frac{1}{2}\%$ amount to \$1049.75?
5. Borrowed \$3500 at $7\frac{1}{2}\%$, and on March 2, 1894, paid \$3850. When was the money borrowed, reckoning time by compound subtraction?
6. How long will it take a sum of money to double itself at 8%?
7. How long must I keep \$937.50 on deposit at 6% interest, to pay a debt of \$975?

PROBLEM IV.

340. Interest, Principal, and Time given, to find Rate.

ILLUSTRATIVE EXAMPLE.

At what rate per cent must \$300 be loaned for 2 yr. 6 mo. to produce \$60 interest?

SOLUTION.

Int. on \$300 for the given time at 1% is \$7.50. $\$60 \div 7.50 = 8\%$. Ans.

EXPLANATION.—Find the interest on \$300 for 2 yr. 6 mo. at 1%, which is \$7.50. Since \$7.50 is the interest at 1%, then \$60 is the interest at such a per cent as \$7.50 is contained times in \$60, or 8%.

CANCELATION.

$$\begin{array}{r|l} 100 & 300 & 3 \\ 4 & 12 & 30 \\ 2 & 60 & ? \end{array}$$

$4 \times 2 = 8\%$. Ans.

EXPLANATION.—Leaving space for rate blank $(100 \times 12 \times 60) \div (300 \times 30) = 8$, the required rate.

EXAMPLES FOR DRILL.

1. At what rate per cent of interest per annum will \$360 produce \$4.80 interest in 60 da.?
2. At what rate will \$1000 yield \$12.50 interest in 2 mo. 15 da.?

3. At what rate per cent per annum will \$3600 amount to \$3902.40 in 1 yr. 2 mo. 12 da.?
4. At what rate per cent of interest will \$3400 produce \$56.10 interest in 66 da.?
5. A debt of \$660 with interest from April 3, 1894 to August 1, 1894, amounts to \$677.60. What was the rate per cent of interest?
6. At what rate per cent will \$500 amount to \$800 in 10 yr.?
7. If a debt of \$7200 in 5 yr. 9 mo. 15 da. amounts to \$9285, what is the rate per cent of interest?

OTHER INTEREST COMPUTATIONS.

341. Time, Rate, and Amount being given, to find the Principal.

ILLUSTRATIVE EXAMPLE.

What principal will, in 1 year 6 months at 6%, amount to \$545?

SOLUTION.

\$1 in 1 yr. 6 mo. at 6% amounts to \$1.09.

$\$545 \div \$1.09 = \$500.$

EXPLANATION.—Since the interest on \$1 for 1 yr. 6 mo. at 6% is \$.09, the amount of \$1 for 1 yr. 6 mo. at 6% is \$1.09, and it will take as many dollars to amount to \$545 as \$1.09 is contained times in \$545, or \$500.

EXAMPLES FOR DRILL.

What principal will amount to —

1. \$677.10 in 4 mo. if loaned at 5%?
2. \$1012 in 3 mo. 6 da. if loaned at $4\frac{1}{2}\%$?
3. What sum of money put at interest for 11 mo. 9 da. at 6%, will amount to \$308.34?
4. A lender receives \$672.10 in payment of a loan with interest at 6% for 3 yr. 8 mo. 12 da. What was the amount of the loan?
5. What principal will amount to \$566.10 in 3 yr. 2 mo. 21 da. at 8%?
6. What sum must I put at interest at 10%, to pay a debt of \$5200 due 3 yr. hence?

ACCURATE INTEREST.

342. Accurate, or Exact Interest is interest which is computed on a basis of 365 days to the year.

343. In ordinary interest a day is considered the $\frac{1}{360}$ of a year, while it is actually only the $\frac{1}{365}$ of a year; therefore, the ordinary interest is $\frac{5}{365}$, or $\frac{1}{73}$ more than the accurate interest; hence, accurate interest may be computed from ordinary interest by subtracting $\frac{1}{73}$ of the latter, and ordinary interest may be obtained from accurate interest by adding $\frac{1}{72}$ of the accurate interest.

ILLUSTRATIVE EXAMPLE.

Find the accurate interest on \$365 for 7 mo. 6 da. at 6%

SOLUTION.—The ordinary interest is \$13.14. Take $\frac{1}{73}$ of this interest, which is \$.18, and subtract it from \$13.14 which leaves \$12.96, the accurate interest.

EXAMPLES FOR DRILL.

Find the accurate interest on —

- | | |
|-------------------------------|--|
| 1. \$184.50 for 15 da. at 5% | 6. \$3140 for 30 da. at 3%. |
| 2. \$240 for 84 da. at 4% | 7. \$5200 for 66 da. at $4\frac{1}{2}\%$. |
| 3. \$396.50 for 24 da at 6% | 8. \$3220 80 for 7 mo. 18 da. 6%. |
| 4. \$384.40 for 124 da. at 6% | 9. \$4500 for 120 da. at 6%. |
| 5. \$368.25 for 75 da. at 7% | 10. \$3610 for 240 da. at $7\frac{1}{2}\%$. |

ANNUAL INTEREST.

344. Annual Interest is the simple interest on the principal, and on each period's simple interest remaining unpaid.

345. Annual, or Periodic Interest is very often secured by a note or series of notes. A note being given for each interest payment and maturing on the date on which the interest payment is due.

To make such interest collectable, the words "with interest payable annually," or "with annual interest," or "with interest payable semi-annually," must be written in the note or contract.

ILLUSTRATIVE EXAMPLE.

What is the interest on \$2000 from May 1, 1890, to March 25, 1895, at 6%, interest payable annually and no payments being made until final settlement?

SOLUTION :	1895 yr. 3 mo. 25 da.	
	1890 " 5 " 1 da.	
	<hr/>	
	4 yr. 10 mo. 24 da.	Time.
	$\$2000 \times \$.06 = \$120$, 1 year's interest.	
1st yr's. interest remained unpaid	3 yr. 10 mo. 24 da.	
2d " " " "	2 " 10 " 24 "	
3d " " " "	1 " 10 " 24 "	
4th " " " "	10 " 24 "	
	<hr/>	
Interest on \$120 for	9 yr. 7 mo. 6 da.	= \$ 69.12
" " \$2000 for	4 " 10 " 24 "	= 588.00
	Total interest due,	<hr/> \$657.12

EXPLANATION.—The time from May 1, 1890, to March 25, 1895, is 4 yr. 10 mo. 24 da. The interest for the 1st year (\$120) was not paid for 3 yr. 10 mo. 24 da. after it was due; the 2d year's interest (\$120) was not paid until 2 yr. 10 mo. 24 da. after it was due; the 3d year's interest (\$120) was not paid until 1 yr. 10 mo. 24 da. after it was due; and the 4th year's interest (\$120) was not paid until 10 mo. 24 da. after it was due; therefore, the total time for which interest should be computed on 1 year's interest (\$120) is 9 yr. 7 mo. 6 da., and the interest is \$69.12; adding to this the interest on the principal for the full time, or 4 yr. 10 mo. 24 da. (\$588), gives the amount of interest due as \$657.12.

EXAMPLES FOR DRILL.

1. Find the interest on \$1200 due in 3 yr. 6 mo. at 6%, with interest payable annually, and no payments having been made.
2. Find the amount due Nov. 19, 1894 on a debt of \$6500 dated July 1, 1892, with interest at 7% payable semi-annually, nothing being paid until final settlement.
3. A note for \$2500 was given Sept. 1, 1891, with interest at 8% payable annually, on which the first two payments were made when due. What was the total amount remaining unpaid Mar. 1, 1895?
4. Find the amount due on a note of \$2000 dated Aug. 1, 1893, and paid Sept. 1, 1894, with interest at 5% payable quarterly, no interest having been paid.

COMPOUND INTEREST.

346. Compound Interest is the interest on the principal alone for the first period, and for subsequent periods on the principal increased by the accrued interest.

ILLUSTRATIVE EXAMPLE.

What is the interest on \$450 for 3 yr. 6 mo. at 6%, if the interest be compounded annually?

SOLUTION.

\$450	principal.
27	1st. year's interest at 6%.
\$477	principal 2d year.
28.62	2d year's interest.
\$505.62	principal 3d year.
30.337	3d year's interest.
\$535.957	principal 4th year.
16.079	6 months' interest.
\$552.036	amount due in 3 yr. 6 mo.
\$450	original principal.
\$102.036	comp. int. for 3 yr. 6 mo.

EXPLANATION.—

Find the first year's interest, which is \$27; add this to the principal, obtaining \$477, the amount due at the end of the first year; on this principal find the interest for the second year and add to principal, obtaining the amount due at the end of the second year, \$505.62; continue this process for the third year; then find the interest on the amount due at the end of the third year for the remaining 6 mo., which is \$16.079; add this to the principal, \$535.957, and the

amount due is found to be \$552.036; then from this amount take the principal, \$450, and the remainder, \$102.036, will be the compound interest.

EXAMPLES FOR DRILL.

What is the compound interest on—

1. \$3000 for 4 yr. with interest at 5%, compounded annually?
2. \$3200 for 3 yr. with interest at 6%, compounded annually?
3. \$2500 for 2 yr. with interest at 8%, compounded semi-annually?
4. \$8000 for 1 yr. with interest at 5%, compounded quarterly?
5. \$350 for 1 yr. 3 mo. with interest at 6%, compounded quarterly?
6. Find the amount due Mar. 13, 1894, on a debt of \$850 with interest at 8%, compounded annually, if the debt is dated Aug. 13, 1889?
7. \$450 was deposited in a savings-bank paying 4% per annum, payable quarterly, and compounded if not withdrawn. What would it amount to at the end of two years?

347. Compound Interest is usually computed by means of an **Interest Table**, showing the amount of \$1 for the various times at different rates. The table upon the following pages shows the amount at compound interest of \$1 for any number of years from 1 to 55, and for any rate from 1% to 10%, which will greatly shorten the work of finding compound interest.

Yrs	1 per ct.	1½ per ct.	2 per ct.	2½ per ct.	3 per ct.	3½ per ct.	4 per ct.	Yrs
1	1.0100 000	1.0150 000	1.0200 000	1.0250 000	1.0300 000	1.0350 000	1.0400 000	1
2	1.0201 000	1.0302 250	1.0404 000	1.0506 250	1.0609 000	1.0712 250	1.0816 000	2
3	1.0303 010	1.0456 784	1.0612 080	1.0768 906	1.0927 200	1.1087 178	1.1248 640	3
4	1.0406 040	1.0613 636	1.0824 321	1.1038 128	1.1255 081	1.1475 290	1.1698 586	4
5	1.0510 101	1.0772 840	1.1040 808	1.1314 082	1.1592 747	1.1876 863	1.2166 529	5
6	1.0615 202	1.0934 433	1.1261 624	1.1596 934	1.1940 520	1.2292 553	1.2653 192	6
7	1.0721 334	1.1098 450	1.1486 857	1.1886 857	1.2298 737	1.2722 792	1.3159 317	7
8	1.0828 567	1.1264 926	1.1716 593	1.2184 020	1.2667 708	1.3168 094	1.2685 690	8
9	1.0936 853	1.1433 900	1.1950 927	1.2488 627	1.3047 718	1.3628 975	1.4233 181	9
10	1.1046 221	1.1605 408	1.2189 942	1.2800 843	1.3439 168	1.4105 987	1.4802 448	10
11	1.1156 683	1.1779 489	1.2433 743	1.3120 806	1.3842 337	1.4599 697	1.5394 546	11
12	1.1268 250	1.1956 182	1.2682 417	1.3448 882	1.4257 608	1.5110 696	1.6010 322	12
13	1.1380 933	1.2135 521	1.2936 063	1.3785 110	1.4685 371	1.5639 568	1.6650 731	13
14	1.1494 742	1.2317 557	1.3194 787	1.4129 738	1.5125 872	1.6186 942	1.7316 764	14
15	1.1609 690	1.2502 321	1.3458 684	1.4482 981	1.5579 674	1.6753 483	1.8009 433	15
16	1.1725 786	1.2689 855	1.3727 857	1.4845 052	1.6047 064	1.7339 801	1.8729 812	16
17	1.1843 044	1.2880 203	1.4002 412	1.5216 126	1.6528 473	1.7946 755	1.9479 005	17
18	1.1961 475	1.3073 406	1.4282 462	1.5506 587	1.7024 336	1.8574 892	2.0258 162	18
19	1.2081 990	1.3269 507	1.4568 117	1.5936 501	1.7535 065	1.9225 012	2.1068 491	19
20	1.2201 900	1.3468 550	1.4859 470	1.6386 164	1.8061 123	1.9897 886	2.1911 234	20
21	1.2323 919	1.3670 578	1.5156 664	1.6795 815	1.8602 947	2.0594 317	2.2787 687	21
22	1.2447 159	1.3875 637	1.5459 767	1.7215 710	1.9161 031	2.1315 158	2.3699 187	22
23	1.2571 630	1.4083 772	1.5768 992	1.7646 108	1.9735 863	2.2061 148	2.4647 155	23
24	1.2697 346	1.4295 028	1.6084 372	1.8087 259	2.0327 911	2.2833 249	2.5633 017	24
25	1.2824 320	1.4509 454	1.6406 059	1.8539 440	2.0937 779	2.3632 448	2.6658 363	25
26	1.2952 563	1.4727 095	1.6784 181	1.9002 927	2.1565 912	2.4459 586	2.7724 697	26
27	1.3082 089	1.4948 002	1.7068 868	1.9478 002	2.2212 891	2.5315 671	2.8833 688	27
28	1.3212 910	1.5172 222	1.7410 212	1.9964 950	2.2879 276	2.6201 719	2.9987 032	28
29	1.3345 039	1.5399 805	1.7758 469	2.0464 073	2.3565 651	2.7118 798	3.1186 515	29
30	1.3478 470	1.5630 802	1.8113 618	1.0975 678	2.4272 624	2.8067 937	3.2433 975	30
31	1.3613 294	1.5865 264	1.8475 882	1.1500 067	2.5000 803	2.9050 318	3.3731 334	31
32	1.3749 407	1.6103 243	1.8845 409	1.2037 569	2.5750 827	3.0067 079	3.5080 587	32
33	1.3886 901	1.6344 792	1.9222 310	1.2588 506	2.6523 352	3.1119 423	3.6483 810	33
34	1.4025 770	1.6589 964	1.9606 703	2.3153 221	2.7319 030	3.2208 603	3.7948 163	34
35	1.4166 028	1.6838 813	1.9998 895	2.3732 051	2.8138 625	3.3335 905	3.9400 889	35
36	1.4307 688	1.7091 395	2.0398 873	2.4325 352	2.8982 783	3.4502 611	4.1039 325	36
37	1.4450 765	1.7347 766	2.0806 850	2.4933 487	2.9852 268	3.5710 254	4.2680 896	37
38	1.4595 272	1.7607 983	2.1222 937	2.5556 824	3.0747 834	3.6960 112	4.4388 134	38
39	1.4741 225	1.7872 103	2.1647 447	2.6195 748	3.1670 268	3.8253 717	4.6183 659	39
40	1.4888 637	1.8140 184	2.2080 396	2.6850 638	3.2620 377	3.9592 592	4.8010 263	40
41	1.5037 521	1.8412 287	2.2522 004	2.7521 904	3.3598 989	4.0978 338	4.9930 615	41
42	1.5187 899	1.8688 471	2.2972 444	2.8209 952	3.4606 959	4.2412 579	5.1927 839	42
43	1.5339 778	1.8968 793	2.3431 896	2.8915 208	3.5645 177	4.3897 020	5.4004 957	43
44	1.5493 176	1.9253 330	2.3900 511	2.9638 080	3.6714 527	4.5433 416	5.6165 158	44
45	1.5648 107	1.9542 130	2.4378 542	2.0379 038	3.7815 958	4.7023 585	5.8411 758	45
46	1.5804 589	1.9835 262	2.4866 112	3.1138 508	3.8950 437	4.8669 410	6.0748 221	46
47	1.5962 634	2.0132 791	2.5363 435	3.1916 913	4.0118 950	5.0372 840	6.3178 156	47
48	1.6122 261	2.0434 783	2.5870 703	3.2714 856	4.1323 518	5.2135 859	6.5705 282	48
49	1.6283 483	2.0741 305	2.6388 117	3.3572 768	4.2562 104	5.3960 645	6.8333 493	49
50	1.6446 318	2.1052 424	2.6915 880	3.4371 087	4.3839 060	5.5849 266	7.1066 835	50
51	1.6610 781	2.1368 211	2.7454 197	3.5230 364	4.5154 232	5.7803 993	7.3909 568	51
52	1.6776 889	2.1688 734	2.8003 281	3.6111 123	4.6508 859	5.9827 137	7.6855 837	52
53	1.6944 658	2.2014 065	2.8563 347	3.7013 901	4.7904 124	6.1921 082	7.9940 526	53
54	1.7114 105	2.2344 276	2.9134 614	3.7939 249	4.9341 243	6.4058 320	8.3138 143	54
55	1.7285 246	2.2679 439	2.9717 267	3.8887 730	5.0821 485	6.6331 414	8.6463 662	55

Yrs	4½ per ct.	5 per ct.	6 per ct.	7 per ct.	8 per ct.	9 per ct.	10 per ct.	Yrs
1	1.0450 0000	1.0500 000	1.0600 000	1.0700 000	1.0800 000	1.0900 000	1.1000 000	1
2	1.0920 2500	1.1025 000	1.1236 000	1.1449 000	1.1664 000	1.1881 000	1.2100 000	2
3	1.1411 6612	1.1576 250	1.1910 160	1.2250 433	1.2597 120	1.2950 290	1.3310 000	3
4	1.1925 1860	1.2155 063	1.2624 770	1.3107 960	1.3604 890	1.4115 816	1.4641 000	4
5	1.2461 8194	1.2762 816	1.3382 256	1.4025 517	1.4693 281	1.5386 240	1.6105 100	5
6	1.3022 6012	1.3400 956	1.4185 191	1.5007 304	1.5868 743	1.6771 001	1.7715 610	6
7	1.3608 6183	1.4071 004	1.5036 303	1.6057 815	1.7138 243	1.8280 391	1.9487 171	7
8	1.4221 0061	1.4774 554	1.5938 481	1.7181 862	1.8509 302	1.9925 626	2.1435 888	8
9	1.4860 9514	1.5513 282	1.6894 790	1.8384 592	1.9990 046	2.1718 993	2.3579 477	9
10	1.5529 6942	1.6288 946	1.7908 477	1.9671 514	2.1589 250	2.3673 637	2.5987 425	10
11	1.6228 5305	1.7103 394	1.8982 986	2.1048 520	2.3316 390	2.5804 264	2.8531 167	11
12	1.6958 8143	1.7958 563	2.0121 965	2.2521 916	2.5181 701	2.8126 648	3.1384 284	12
13	1.7721 9610	1.8856 491	2.1329 283	2.4098 450	2.7196 237	3.0658 046	3.4522 712	13
14	1.8519 4492	1.9799 316	2.2609 040	2.5785 342	2.9371 936	3.3417 270	3.7974 983	14
15	1.9352 8244	2.0789 282	2.3965 582	2.7590 315	3.1721 691	3.6424 825	4.1772 482	15
16	2.0223 7015	2.1828 746	2.5403 517	2.9521 638	3.4259 426	3.9703 059	4.5949 730	16
17	2.1133 7681	2.2920 183	2.6927 728	3.1588 152	3.7000 181	4.3276 834	5.0544 703	17
18	2.2084 7877	2.4066 192	2.8543 392	3.3799 323	3.9960 195	4.7171 204	5.5599 173	18
19	2.3078 6031	2.5269 502	3.0255 995	3.6165 275	4.3157 011	5.1416 613	6.1159 390	19
20	2.4117 1402	2.6532 977	3.2071 355	3.8696 845	4.6609 571	5.6044 108	6.7275 000	20
21	2.5202 4116	2.7839 626	3.3995 636	4.1405 624	5.0338 337	6.1088 077	7.4002 499	21
22	2.6336 5201	2.9252 607	3.6035 374	4.4304 017	5.4365 404	6.6586 004	8.1402 749	22
23	2.7521 6635	3.0715 238	3.8197 497	4.7405 299	5.8714 637	7.2578 745	8.9543 024	23
24	2.8760 1383	3.2250 999	4.0489 346	5.0723 670	6.3411 807	7.9110 832	9.8497 327	24
25	3.0054 3446	3.3863 549	4.2918 707	5.4274 326	6.8484 752	8.6230 807	10.8347 059	25
26	3.1406 7901	3.5556 727	4.5493 839	5.8073 529	7.3963 532	9.3991 579	11.9181 765	26
27	3.2820 0956	3.7334 563	4.8223 459	6.2138 676	7.9880 615	10.2450 821	13.1099 942	27
28	3.4296 9999	3.9201 291	5.1116 867	6.6488 384	8.6271 064	11.1671 395	14.4209 936	28
29	3.5840 3649	4.1161 356	5.4183 879	7.1142 571	9.3172 749	12.1721 821	15.8630 930	29
30	3.7453 1813	4.3219 424	5.7434 912	7.6122 550	10.0626 569	13.2676 785	17.4494 023	30
31	3.9138 5745	4.5380 395	6.0881 006	8.1451 129	10.8676 694	14.4617 935	19.1943 425	31
32	4.0899 8104	4.7649 415	6.4533 867	8.7152 708	11.7370 830	15.7633 288	21.1137 768	32
33	4.2740 8018	5.0031 885	6.8405 899	9.3253 398	12.6760 496	17.1320 284	23.2251 544	33
34	4.4663 6154	5.2533 480	7.2510 253	9.9781 135	13.6901 336	18.7284 109	25.5476 699	34
35	4.6673 4781	5.5160 154	7.6860 868	10.6765 815	14.7853 443	20.4139 679	28.1024 369	35
36	4.8773 7846	5.7918 161	8.1472 520	11.4239 422	15.9681 718	22.2512 250	30.9126 805	36
37	5.0968 6049	6.0814 069	8.6360 871	12.2236 181	17.2456 256	24.2538 353	34.0039 486	37
38	5.3262 1921	6.3854 773	9.1542 524	13.0792 714	18.6252 756	26.4366 805	37.4043 434	38
39	5.5658 9903	6.7047 512	9.7035 075	13.9948 204	20.1152 977	28.8159 817	41.1447 778	39
40	5.8163 6454	7.0399 887	10.2857 179	14.9744 578	21.7245 215	31.4094 200	45.2592 556	40
41	6.0781 0094	7.3919 882	10.9028 610	16.0226 699	23.4624 832	34.2362 679	49.7851 811	41
42	6.3516 1548	7.7615 876	11.5570 327	17.1442 568	25.3394 819	37.3175 320	54.7636 992	42
43	6.6374 3818	8.1496 639	12.2504 546	18.3443 548	27.3666 404	40.6761 098	60.2400 692	43
44	6.9361 2290	8.5571 503	12.9854 819	19.6284 596	29.5539 717	44.3369 597	66.2640 761	44
45	7.2482 4843	8.9850 078	13.7646 108	21.0024 518	31.9204 494	48.3272 861	72.8904 837	45
46	7.5744 1961	9.4342 582	14.5904 875	22.4726 234	34.4740 853	52.6767 419	80.1795 321	46
47	7.9152 6849	9.9059 711	15.4659 167	24.0457 070	37.2320 122	57.4176 486	88.1974 853	47
48	8.2714 5357	10.4012 697	16.3938 717	25.7289 065	40.2105 791	62.5852 370	97.0172 338	48
49	8.6436 7107	10.9213 331	17.3775 040	27.5298 303	43.4274 130	68.2179 083	106.7189 572	49
50	9.0326 3627	11.4673 998	18.4201 543	29.4570 251	46.9016 125	74.3575 201	117.3908 529	50
51	9.4391 0490	12.0407 698	19.5253 635	31.5190 168	50.6337 415	81.0496 923	129.1299 382	51
52	9.8638 6463	12.6428 083	20.6968 853	32.7252 459	54.7060 408	88.3441 696	142.0422 326	52
53	10.3077 3853	13.2749 487	21.9386 985	36.0861 224	59.0825 241	96.2951 449	156.2472 232	53
54	10.7715 8677	13.9386 961	23.2550 204	38.6121 509	63.8091 260	104.9617 079	171.8719 477	54
55	11.2563 0817	14.6356 309	24.6503 216	41.3150 015	68.9133 561	114.4082 616	189.0591 425	55

348. The amount for any number of years not given in the table may be ascertained by finding the product of the amount of \$1 for any two or more numbers of years whose sum is equal to the given time; thus, the amount of \$1 at compound interest for 70 yr. at 6% is equal to the product of the amount of \$1 at 6% for 50 yr. (\$18.4201543) multiplied by the amount of \$1 at 6% for 20 yr. (\$3.2071355), that is, $\$18.4201543 \times \3.2071355 .

If interest is to be compounded semi-annually take $\frac{1}{2}$ the rate for twice the time.

If interest is to be compounded quarterly take $\frac{1}{4}$ the rate for four times the time.

349. To find the compound interest by the use of the tables.

ILLUSTRATIVE EXAMPLE.

What is the interest on \$4000 for 10 yr. compounded annually at 6%?

SOLUTION.

\$1.7908477 Amount of \$1 for 10 yr. at 6%.

\$1 Principal.

\$.7908477 Comp. Int. on \$1 for 10 yr. at 6%.

\$ 4000 Principal.

\$3163.3908 Comp. Int. on \$4000 for 10 yr. at 6%.

principal from this amount, which leaves \$.7908477 compound interest on \$1 for 10 yr. at 6%; then, the compound interest on \$4000 will be 4000 times the compound interest on \$1, or $\$.7908477 \times 4000 = \3163.39 .

EXPLANATION.—By consulting the interest tables, it is found that the amount of \$1 for 10 yr. at 6% is \$1.7908477; then subtract the original

EXAMPLES FOR DRILL.

1. What is the interest on \$300 for 15 yr., compounded annually at 7%?
2. Find the interest on \$900 for 10 yr. with interest compounded annually at 4%?
3. What is the amount of \$3000 for 26 yr. compounded annually at 5%?
4. What will \$4400 amount to in 10 yr. 6 mo. if the interest be compounded yearly at 8%?
5. What is the interest on \$1200 for 15 yr., if compounded half-yearly at 6%?

6. What is the compound interest on \$3600 for 26 yr. 4 mo. at 8%, payable annually?

350. To find the principal of an amount at compound interest.

EXAMPLE.—What principal will in 2 yr. at 8% amount to \$847.80?

EXPLANATION.—By the table \$1 in 2 yr. at 8% amounts to \$1.1664; then it will take as many dollars to amount to \$847.80 as \$1.1664 is contained times in \$847.80, or \$750.

EXAMPLES FOR DRILL.

1. What principal will amount to \$2590.71 in 10 yr. with interest compounded annually at 8%?

2. What sum will yield \$133.39 compound interest in 4 yr., at 6%, if compounded semi-annually?

3. What principal must be loaned for 3 yr., with interest at 8% compounded quarterly, to amount to \$1268.24?

BUSINESS PAPER.

351. Business, or Commercial Paper includes notes, checks, drafts, bills of exchange, etc.

352. A Negotiable Paper is any written instrument representing a certain amount of money due at some certain time, which may be transferred from one party to another, and which vests the absolute right of collection in the party to whom it is transferred.

353. Non-Negotiable Paper is not considered transferable, but is assignable, and the party to whom it is assigned may collect it.

354. A Note is a promise to pay a specified sum of money to a certain person on demand, or at a specified time.

355. The Maker of a note is the person who makes or writes the note and whose name is signed to it.

356. The Payee is the party to whom it is made payable.

357. The Face of a note is the amount for which it is given.

358. A Joint Note is a note signed by two or more persons, who are jointly liable for its payment.

359. A Joint and Several Note is a note made by several parties, who are held individually responsible for the whole amount.

Notes payable to bearer are transferable by delivery.

Notes payable to a certain person or order may be transferred by indorsement and delivery.

360. An Indorsement is anything written on the back of a note or other business paper, and is made for three purposes:—

1. To secure the payment.
2. To transfer the title.
3. To make a memorandum of partial payments.

The person who indorses a note is called the **indorser**, and the person in whose favor the indorsement is made is called the **indorsee**.

361. An Indorsement in blank is simply the signature of the indorser written on the back of the paper, and after such an indorsement it is payable to any person who may subsequently become its holder.

Any legal holder of a note indorsed in blank is allowed to write words above his name that will make it an indorsement in full.

362. An Indorsement in full is one in which the indorser states over his signature the person to whose order the instrument is payable.

363. A Qualified Indorsement is one in which the indorser relieves himself from responsibility by writing above his signature the words "without recourse." By writing these words the indorser relieves himself from all liability for the payment of the note.

364. A Restrictive Indorsement is one which restricts the payment to a certain person.

365. Days of Grace are three days allowed by law for the payment of a note, after the time specified in the contract; they are usually allowed on all time paper, but some states have abolished them entirely.

366. The Maturity of a note is the time at which it is legally due, and when days of grace are allowed, the note matures on the last day of grace. If the last day of grace falls on Sunday or on a legal holiday, in most states it must be paid on the preceding day, or second

day of grace, and if the second day of grace should be a legal holiday, it matures on the first day of grace.

If a note is made payable in a certain number of days, the date of maturity is determined by counting on from the date of the note, the expressed number of days plus the days of grace.

If the note is given for so many months, the date of maturity is determined by adding to the date of the note the number of months for which it is given, plus the days of grace, regardless of the number of days in a month. Thus, a note dated May 31, due one month after date, would be nominally due on the last day of June, or June 30; or notes dated Jan. 28, 29, 30, 31, payable in one month, would be nominally due on the last day of February, or Feb. 28, and legally due March 3.

If a note is to draw interest, it should be so stated in the note, using the words "with interest" or "with use," otherwise it will not bear interest until after maturity; but according to a recent decision in the State of Michigan, a demand note does not bear interest until after demand has been made, even when these words are written in the note.

Demand for the payment of a note should be made on the date of maturity and upon refusal of the maker to pay the note, it should be at once protested and notice sent to the indorsers on the day of maturity, or not later than the next day; otherwise, they cannot be held responsible.

367. A Protest of a negotiable note is a formal declaration of the demand and refusal to pay.

In case of non-payment of the note by the maker, each indorser can be held liable only after receiving a sufficient notice.

What would be considered a sufficient notice varies in the different states; as in Michigan, a written notice of demand and non-payment by the payee himself would be sufficient, while in other states it would be required that this notice be written by a notary public.

PARTIAL PAYMENTS.

368. Partial Payments are payments in part on a note or other obligation.

Partial payments of a note or mortgage are usually acknowledged by indorsing the amount paid and the date on the back of the instrument, and very often special receipts are given for the payment.

When partial payments are made on an interest-bearing debt, the amount due at any specified time is usually found by the following method, which has been adopted by the Supreme Court of the United States, and is called the *United States Rule*.

369. Payments must be applied first to the discharge of the accrued interest, and then the remainder, if any, toward the discharge of the principal.

UNITED STATES RULE FOR PARTIAL PAYMENTS.

ILLUSTRATIVE EXAMPLE.

Find the amount due Apr. 15, 1895, on a note of \$4000 dated Mar. 12, 1892, drawing interest at 6%, on which are indorsed the following payments : —

Sept. 12, 1892,	\$620;	Dec. 6, 1893,	\$50;
Feb. 18, 1893,	\$1441;	Jan. 30, 1894,	\$1000.

Find time by compound subtraction.

SOLUTION.

Face of note,	\$4000.00
Int. from date of note (Mar. 12) to first payment (Sept. 12) 6 mo.,	120.00
Amount due at time of first payment,	4120.00
First payment to be deducted,	620.00
Debt remaining unpaid Sept. 12, 1892,	3500.00
Int. from date of first payment (Sept. 12, 1892) to second payment (Feb. 18, 1893) 5 mo., 6 da.,	91.00
Amt. due at time of second payment (Feb. 18, 1893),	3591.00
Second payment deducted,	1441.00
Debt remaining unpaid Feb. 18, 1893,	2150.00
Int. from date of second payment (Feb. 18, 1893) to date of third payment (Dec. 6, 1893),	103.20
The interest being larger than the payment on this date, in- terest is not added nor payment deducted.	
Int. from date of third payment (Dec. 6, 1893) to date of fourth payment (Jan. 30 1894). 1 mo 24 da.	19.35

PARTIAL PAYMENTS.

173

Amount due at time of fourth payment,	2272.55
Third and fourth payments deducted,	1050.00
Debt remaining unpaid Jan. 30, 1894,	<u>1222.55</u>
Int. from date of fourth payment (Jan. 30, 1894) to date of settlement (Apr. 15, 1895) 1 yr. 2 mo. 15 da.,	88.63
Amount due at date of settlement,	<u>\$1311.18</u>

NOTE.—Payments are not deducted unless they exceed the accrued interest at date of payment, but are held until the sum of the payments made is larger than the accrued interest.

In the above solution it could be easily seen that the third payment would be less than the interest due at the date of the payment, and the interest could have been more easily computed in one operation from the date of the preceding payment (Feb. 18, 1893) to the date of the fourth payment (Jan. 30, 1894) or for 11 months 12 days, obtaining \$122.55; the same interest in the solution was obtained in the two operations, $\$103.20 + \$19.35 = \$122.55$.

RULE.—*Find the amount of the principal to the time when the payment, or the sum of the payments, is equal to or greater than the interest then due; from the amount thus obtained subtract the payment or the sum of the payments made, and treat the remainder as a new principal; proceed in this manner to the date of settlement, and the last amount will be the sum still due.*

NOTE.—In applying the above rule to partial payments, find the time by compound subtraction.

EXAMPLES FOR DRILL.

1.

\$1200.

Battle Creek, Mich., Jan. 15, 1890.

On demand I promise to pay to the order of L. F. Hall, One Thousand Two Hundred $\frac{00}{100}$ Dollars, with interest at 6% per annum from date.
J. VAN ALLEN.

On the above note were the following indorsements:—

July 15, 1890,	\$125,	Sept. 15, 1891,	\$400;
Dec. 30, 1890,	\$250;	July 30, 1893,	\$300.

Find the amount due July 15, 1894.

2.

\$350.

Buffalo, N. Y., Mar. 17, 1891.

One year after date, for value received, I promise to pay L. H. Beach, or order, Three Hundred Fifty $\frac{00}{100}$ Dollars, with interest at 8% per annum.

W. S. WATSON.

Indorsed: Mar. 17, 1892, \$50;

Nov. 15, 1892, \$100;

July 1, 1894, \$20.

What was the balance due Jan. 17, 1895?

3.

\$750.

Chicago, Ill., Aug. 12, 1890.

Three years after date we promise to pay to the order of Arthur Shaw, Seven Hundred Fifty $\frac{00}{100}$ Dollars, with interest at 6% per annum.

GEO. ELLIOTT & Co.

The following payments were indorsed on the above note: —

Jan. 18, 1891, \$75;

Apr. 24, 1892, \$150.

Dec. 15, 1891, \$10;

Nov. 12, 1892, \$15;

May 18, 1893, \$450.

Find the amount due Aug. 15, 1893.

4.

\$3500.

St. Louis, Mo., Aug. 15, 1892.

One year after date I promise to pay H. K. Thomas, or order, Thirty five Hundred $\frac{00}{100}$ Dollars, with interest at 7% per annum.

J. E. CONNELLY.

The above note was indorsed as follows: —

May 18, 1893, \$1000;

Apr. 16, 1894, \$350;

Dec. 24, 1893, \$100;

Nov. 24, 1894, \$50.

What was due Mar. 8, 1895?

5.

On a mortgage for \$6000 dated July 15, 1888, and bearing interest at 8% per annum, the following payments were made: Sept. 30, 1889, \$600; Oct. 15, 1890, \$1500; Sept. 25, 1891, \$150; July 15, 1893, \$1000. What was the amount due July 15, 1894?

6.

A mortgage was given, bearing date of Nov. 1, 1886, for \$3000, drawing interest at 6%, on which the following payments were made: July 16, 1888, \$1500; Nov. 17, 1889, \$125; Sept. 18, 1891, \$350; July 12, 1892, \$700; Aug. 16, 1894, \$200.

Find the balance due Mar. 16, 1895.

MERCHANTS' RULE.

370. The Commercial, or Merchants' Rule for partial payments is commonly employed by business men when the date of settlement is one year or less from the date of a note or account bearing interest.

ILLUSTRATIVE EXAMPLE.

A note of \$800 was given Sept. 15, 1894, bearing interest at 6%, on which were indorsed the following payments: Oct. 27, 1894, \$50; Dec. 17, 1894, \$100; Mar. 8, 1895, \$200.

Find the balance due Apr. 25, 1895.

SOLUTION.

Face of note dated Sept. 15, 1894,		\$800.00
Interest on \$800 from Sept. 15, 1894, to Apr. 25, 1895, 222 days,		29.60
Amount of principal and accrued interest at date of settlement,		<u>\$829.60</u>
First payment made Oct. 27, 1894,	\$ 50.00	
Interest on \$50 from Oct. 27, 1894, to Apr. 25, 1895, 180 days,	1.50	
Second payment made Dec. 17, 1894,	100.00	
Interest on \$100 from Dec. 17, 1894, to Apr. 25, 1895, 129 days,	2.15	
Third payment made Mar. 8, 1895,	200.00	
Interest on \$200 from Mar. 8, 1895 to Apr. 25, 1895, 48 days,	<u>1.60</u>	
Sum of payments and accrued interest at settlement,		355.25
Balance due at date of settlement,		<u>\$474.35</u>

NOTE.—In the solution of problems by the Merchants' Rule, it is customary to find the exact time in days.

As will be seen by the preceding solution, interest is allowed on the original amount of the debt from date incurred to date of settlement, and then interest is allowed on each payment from the time made to date of settlement.

It is customary among business men, on notes or other accounts running longer than one year, to find the balance due at the end of the first year, after deducting all payments, and consider this balance a new principal; and in the same way find the balance due at the end of the second year, and so on, until the debt is paid in full. The times at which these balances are made are called **Rests**. In some cases banks have been allowed to make **Quarterly Rests**.

RULE.—*Find the amount of the principal to the date of settlement; find the interest on each payment from the time it was given to the date of settlement; then subtract the sum of the payments and the interest thereon from the amount of the principal.*

EXAMPLES FOR DRILL.

1. What is the balance due May 16, 1894, on a note for \$800, with interest at 6%, dated Sept. 15, 1893, on which the following indorsements were made: Nov. 25, 1893, \$100; Jan. 16, 1894, \$250; Mar. 18, 1894, \$300?

2. On a note of \$1000, at 5%, dated Jan. 5, 1893, were indorsed the following payments: Mar. 15, 1893, \$300; Apr. 25, 1893, \$200; July 16, 1893, \$400. Find the balance due Sept. 5, 1893.

3. What was the balance due Oct. 18, 1894, on a note of \$1824, at 8%, dated Jan. 4, 1894, on which the following indorsements were made: Mar. 3, 1894, \$200; Apr. 18, 1894, \$100; June 15, 1894, \$350; Aug. 20, 1894, \$450; Sept. 18, 1894, \$200?

4. If the balance due on the preceding note, Oct. 18, 1894, is not paid, what would be the amount due one year from that date, no payments having been made during the interval?

5. A settled his account by \$750 demand note bearing 12%, on interest Mar. 3, 1894. Payments, Sept. 5, 1894, \$75; Jan. 7, 1895, \$250. What is due Oct. 6, 1895?

6.

\$1500.

Detroit, Mich., Mar. 3, 1892.

One year after date I promise to pay C. Reasoner, or order, Fifteen Hundred $\frac{00}{100}$ Dollars, with interest at 8% per annum.

W. S. TAYLOR.

The following payments were made on the above note : —

June 3, 1892,	\$150;	Nov. 3, 1892,	\$300;
Sept. 18, 1892,	\$200;	Jan. 18, 1893,	\$500.

Find balance due one year from date, or Mar. 3, 1893.

7. The balance due March 3, 1893, on the foregoing note, not being paid, find the balance due at the end of the next year, or Mar. 3, 1894, if the following payments were made during that time : Sept. 12, 1893, \$100; Dec. 18, 1893, \$250.

8. On Aug. 18, 1893, I gave my note of \$2000, at 6%, upon which I afterward made the following payments : Nov. 3, 1893, \$500; Jan. 15, 1894, \$300; Mar. 18, 1894, \$150; May 15, 1894, \$75. What did I owe on the note July 18, 1894?

TRUE DISCOUNT.

371. Discount is the allowance made for the payment of a debt before it is legally due.

372. The Present Worth of a debt due at a future time is its value now. Hence, it is a sum which, if put at interest to-day, will amount to the debt, when that debt becomes legally due.

373. True Discount is the difference between the present worth of a debt and its value when legally due, or the simple interest on the present worth of a debt for the time until the debt is due.

ILLUSTRATIVE EXAMPLE.

A debt of \$700 is legally due 1 year 6 months from to-day. What is the present worth of the debt, if money is worth 8%? What discount should be allowed if paid to-day?

SOLUTION.

\$1.12 = amount of \$1 for 1 yr. 6 mo. at 8%.

\$700 ÷ \$1.12 = \$625 present worth.

\$700 — \$625 = \$75 true discount.

EXPLANATION.— The interest on \$1 for 1 yr. 6 mo. at 8% is \$.12; the amount of \$1 for 1 yr. 6 mo. is \$1.12; therefore, the present worth of a debt of \$1.12 due in 1 yr. 6 mo. without interest is \$1; hence the present worth of \$700 will be as many dollars as \$1.12 is contained times in \$700, or \$625.

NOTE.— To find the present worth of a debt bearing interest, first find what the debt would amount to at maturity, and then find the present worth of this amount at the time the debt is discounted.

EXAMPLES FOR DRILL.

Find the present worth and true discount of —

1. \$345 due in 2 yr. 6 mo., money being worth 6%.
2. \$605 due in 1 yr. 8 mo., money being worth 6%.
3. \$364.80 due in 60 da., money being worth 8%.
4. \$507.75 due in 93 da., money being worth 6%.
5. \$345.61 due in 66 da., money being worth 9%.
6. Which is the cheaper and how much per barrel, to buy flour at \$5 a barrel on 60 days' credit, or at \$5.25 on 3 months' credit, money being worth 8%?
7. A wholesale merchant offered an invoice of goods for \$1035.25 on 60 days' credit. What would be an equivalent cash offer, money being worth 6%?
8. A horse was bought for \$150 cash and on the same day it was sold for \$164 on 8 months' time. If money is worth 8% what was the gain or loss?

SUGGESTION.— Compare the cost with the present worth of the selling-price.

9. A merchant bought an invoice of carpets at \$1.75 per yard on 6 months' time, and sold it on the same day for \$1.60 per yard, cash. If money is worth 6%, what was the gain or loss on 1000 yds. of carpet?

10. I bought an invoice of goods amounting to \$2400 on 60 days' time, on which I was allowed a discount of 10% and 10%; in 6 days I paid the present worth of the debt on that date. Money being worth 6%, how much less than the invoice-price did I pay?

11. A house was bought for \$3500 cash, and immediately sold for \$4000, to be paid in 1 yr. and 8 mo. without interest. Find the gain or loss, if money was worth 8% per annum.

12. How much will I lose by borrowing money at 8% to pay a debt of \$4000 due 2 yr. hence, if by paying cash I could get a discount of 10% from the face of the bill?

13. A real estate agent sold a lot for \$3600, payable in three equal instalments without interest, in 30, 60, and 90 days, respectively. What would be an equivalent cash sale, money being worth 6% per annum?

14. A stock of goods was invoiced at \$1500, and offered for sale at 10% and 12% off for cash; \$1220 on 4 months' credit; \$1250 on 8 months' credit; \$1275 on 1 year's credit. Which is the best offer, and how much better than the next best, money being worth 6%?

BANK DISCOUNT.

374. Bank Discount is a deduction made from the amount of a negotiable paper, by a person who cashes such paper before it is due.

375. The Term of Discount is the time from the day of discounting until the paper falls due (plus three days in the states allowing days of grace).

376. Bank Discount is computed by taking the simple interest on the amount of the bill for the Period of Discount.

377. The Proceeds is the sum paid by the buyer of the bill and is equal to the face of the bill less the Bank Discount.

NOTE.—In finding the Period of Discount, count the exact number of days.

378. To find the Bank Discount and Proceeds of a Note.

ILLUSTRATIVE EXAMPLE.

A note of \$560, dated May 15, 1894, and payable 5 months after date, was discounted at a bank at 6%, Aug. 25, 1894. Find the date of maturity, term of discount, the bank discount, and the proceeds of the note.

SOLUTION.

May 15 + (5 mo. 3 da.) = Oct. 18, date of maturity.

Aug. 25 to Oct. 18 = 54 da., term of discount.

Int. on \$560 for 54 da. at 6% = \$5.04, bank discount.

\$560 — \$5.04 = \$554.96, proceeds.

EXPLANATION.—The date of maturity will be 5 mo. and 3 da. from May 15, or Oct. 18; then find time from date of discount (Aug. 25), to date of maturity

(Oct. 18), which is 54 days, the time for which the note is to be discounted. Compute the interest on \$560 for 54 da. at 6% which gives \$5.04, the bank discount, and the proceeds will be \$560 — \$5.04, or \$554.96.

NOTE.— When interest is mentioned and no rate given, use 6%.

EXAMPLES FOR DRILL.

Find date of maturity, term of discount, bank discount, and proceeds—

1. Of a note for \$940, dated Sept. 12, 1890, given for 90 days and discounted Oct. 15, 1890, at 6%.

2. Of a note for \$340, dated July 7, 1891, given for 60 days, and discounted Aug. 21, 1891, at 8%.

3. Of a note for \$1200, dated Feb. 1, 1892, given for 90 days, and discounted Mar. 25, 1892, at 5%.

4. Of a note for \$3600, dated July 23, 1893, given for 3 months, and discounted Sept. 15, 1893, at 8%.

5. Of a note for \$348, dated Apr. 2, 1894, given for 1 year, and discounted Jan. 15, 1895, at 7%.

6. Of a note for \$1500, dated Aug. 2, 1894, given for 90 days, and discounted Oct. 13, 1894, at 6%.

7. What is the bank discount on a note for \$2500, discounted at a bank for 33 days, at 6%?

8. Find the bank discount and proceeds of the following note:—

\$340.

Detroit, Mich., Oct. 24, 1894.

Sixty days after date I promise to pay to the order of L. L. Dennison, Three Hundred Forty $\frac{00}{100}$ Dollars.

Discounted Dec. 20, 1894, at 6%.

J. L. JOHNSON.

9. What would be the proceeds and discount of the following note if discounted at the bank, Sept. 10, 1893, at 8%?

\$840.

New York, N. Y., July 1, 1893.

Three months after date I promise to pay to H. L. Robbins, or order, Eight Hundred Forty $\frac{00}{100}$ Dollars.

W. L. ROGERS.

NOTE.— Days of grace have been abolished in New York.

10. Find the proceeds of the following note if discounted July 25, 1894, at 8%.

\$560.

Chicago, Ill., May 23, 1894.

Ninety days after date we promise to pay J. B. Saunders, or order,
Five Hundred Sixty $\frac{00}{100}$ Dollars, with interest at 7%.

J. C. AUSTIN.

B. H. DURAND.

NOTE.—Find the amount due at maturity, then discount this amount for
the term of discount. No days of grace.

11. Find the maturity, term of discount, and proceeds of the follow-
ing note :—

\$3600.

Battle Creek, Mich., Dec. 24, 1894.

Four months after date we jointly and severally promise to pay to
the order of H. J. King, Three Thousand Six Hundred $\frac{00}{100}$ Dollars, with
use after one month.

Discounted Feb. 24, 1895, at 8%.

H. D. FERGUSON.

C. E. EDWARDS.

12. Find the discount on the following note, discounted Jan. 15,
1895, at 7%.

\$650 $\frac{00}{100}$.

Boston, Mass., Jan. 2, 1895.

Thirty days after date we promise to pay to the order of Jas. J.
Hewett, Six Hundred Fifty $\frac{00}{100}$ Dollars, with interest at 5%.

PEAT & Co.

13. Find the time to run, and proceeds, of the following note, dis-
counted Dec. 22, 1894, at 6%.

\$375 $\frac{80}{100}$.

Philadelphia, Pa., Oct. 26, 1894.

For value received six months after date I promise to pay F. H.
Clark, or order, Three Hundred Seventy-five $\frac{80}{100}$ Dollars, with interest at
6%, without defalcation.

G. H. BLISS.

14. On Jan. 2, 1895, R. H. Eldred had the following notes dis-
counted at a bank at 8%. Find the total proceeds.

No. 1, dated Dec. 28, 1894, payable 2 mo. after date, \$1200.

No. 2, dated Dec. 29, 1894, payable 2 mo. after date, \$600.

No. 3, dated Dec. 30, 1894, payable 2 mo. after date, \$76.

No. 4, dated Dec. 31, 1894, payable 2 mo. after date, \$750.

NOTE.—Observe that each note matures legally upon the same date,
Mar. 3, 1895.

379. To find the Face of a Note, which when discounted will yield a stated Proceeds.

ILLUSTRATIVE EXAMPLE.

For what sum must a note be given, payable in 60 days that when discounted at 6% the proceeds may be \$791 60?

SOLUTION.

\$1.00	Face of note for \$1.
0105	Discount on \$1 for 63 days.
<hr/> \$.9895	Proceeds of note of \$1
$\$791.60 \div .9895 = \$800.$ Face of note required.	

EXPLANATION.—The discount on a note for 63 days whose face is \$1, is \$.0105; the proceeds would be \$1 — \$.0105, or \$.9895; then, if the proceeds of a note whose face is \$1 is \$.9895, to produce \$791.60 proceeds, a note must be drawn for as many dollars as \$.9895 is contained times in \$791.60, or \$800.

EXAMPLES FOR DRILL.

1. For what amount must a note be given at 60 days, to produce \$534.33, when discounted at 6%?

2. What is the face of a 90-day note, which when discounted at 10% will produce \$900?

3. I wish to borrow \$1000 from a bank. For what sum must I give my note at 30 days, if discounted at 8%?

4. Find the face of a note given for 3 months, that when discounted at the bank at 6%, will produce \$2000.

5. What must be the face of a note dated Nov. 4 1894, and payable 3 months after date, that when discounted Jan. 8, 1895, at 7%, the proceeds will exactly pay a debt of \$500?

6. A depositor had overdrawn his bank account \$325, and to make his account good, he gave the bank a note of \$400, which was legally due in 24 days, and was accepted at a discount of 8%; he also gave the bank his own note for such an amount, due in 60 days, that when discounted at the above rate and the proceeds placed to his credit, he had \$300 to his credit at the bank. What was the face of the second note?

COMMISSION.

380. Commission is the sum charged by an agent for buying or selling property, or for collecting debts.

381. An Agent is a person who transacts business of any kind for another.

382. The Principal is the person for whom the business is transacted.

NOTE.—Persons who do business on commission are variously called Commission merchants, brokers, factors, collectors, etc., according to the character of the business.

383. A Consignment is a shipment of goods from one person to another to be sold on account of the shipper. The shipper is called the *Consignor*, and the person to whom the goods are shipped is called the *Consignee*, or sometimes the *Correspondent*.

384. The Gross Proceeds of a Consignment is the total amount received for it by the agent before deducting the commission, or other charges.

385. The Net Proceeds is the amount due the consignor from the consignee, after deducting the commission and all other charges.

386. The Prime, or Net Cost of a consignment is the sum expended by an agent for the purchase of goods.

387. The Gross Cost is the *Prime Cost plus the Commission* and all other charges arising from the purchase of a consignment.

388. An Account Sales is an itemized statement rendered by the consignee or agent, to the consignor or principal, showing in detail the sales, charges and other expenses, and the net proceeds.

389. An Account Purchase is an itemized statement rendered by the agent to his principal, showing the quantity, grade, and the price of the goods bought, all expenses connected therewith, and the gross amount of the purchase.

390. Guaranty is an agreement made by a commission merchant to become responsible for the payment of goods sold on credit. An extra percentage is usually charged for guaranty.

391. Commissions are computed as percentages upon the amount of business done by the agent. Commissions are not allowed, however,

upon sums spent by an agent for freight charges, insurance and other expenses incident to the purchase or sale.

392. The *Base in Commission*, is the *Gross Proceeds*, the *Prime*, or *Net Cost*, and in collection business the *Amount Collected*.

The *Percentage* is the amount of commission.

EXAMPLES FOR DRILL

1. An agent sold goods to the amount of \$3500, on a commission of $2\frac{1}{2}\%$. What was the amount of the commission?

SOLUTION.

$$\$3500 \times 2\frac{1}{2}\% = \$87.50, \text{ amount of commission.}$$

EXPLANATION.—Take $2\frac{1}{2}\%$ of the gross proceeds, \$3500.

NOTE.—As $2\frac{1}{2}\%$ is $\frac{1}{4}$ of 10%, shorten the work by dividing \$350 (10% of \$3500) by 4 = \$87.50.

2. If an agent's commission is 2%, what will be the amount of the commission for selling property valued at \$13000?

3. A commission merchant sold 300 bbl. of flour at \$5.25 per barrel, on a commission of $3\frac{1}{4}\%$. Find the commission and net proceeds.

4. A collector received a claim for \$3400, which he collected and deducted his commission of $3\frac{1}{2}\%$. How much should he remit and what was the amount of his commission?

5. Frank A. Logan & Co., of Chicago, sold Thompson's consignment for \$1255 at 5% commission. What sum should Logan & Co. remit to Thompson?

6. A real estate agent sold a farm of 160 acres, at \$56 per acre, on a commission of 2%; the stock and farming implements for \$2560, on a commission of 7%. What was the amount of his commission?

7. An architect charges $\frac{1}{8}\%$ for plans and specifications of a building, and $2\frac{1}{4}\%$ for superintending the work. What would the architect receive, if the building cost \$9840?

8. An agent bought for Marshall Field, of Chicago, a carriage for \$780, a pair of horses for \$520, and harness for \$110, and paid charges for keeping, shipping, etc. \$25.25, freight \$56. What was his commission at $3\frac{1}{8}\%$, and what was the total amount of the bill?

9. On sales amounting to \$950, an agent's commission was \$47.50. What was the rate of commission?

10. A real estate agent sold two houses for \$14250, on which the commission was \$926.25. Find rate per cent of commission.

11. If the sales were \$5280, and the proceeds \$5068.80, what was the rate per cent of commission?

12. Bought 10000 bu. of wheat at 52¢ a bushel, and paid \$120 for freight. The gross cost was \$5385. What rate per cent was charged for commission?

13. Field & Co. sold a consignment of fruit, and remitted as net proceeds \$727.65, charging \$18.90 commission, and \$9.45 for guaranteeing payment. What was the rate of commission and guaranty?

14. Received \$12.75 for collecting a claim at a commission of 5%. What was the amount of the claim?

15. If the commission was \$187.25, and the rate of commission 7%, what was the amount of the sales?

16. Find the amount of sales if the commission is \$25.26 and the rate $4\frac{1}{2}\%$.

17. A commission merchant charged \$259.64 for selling a consignment. If the rate of commission was 8%, what must have been the total sales?

18. A collector charged $6\frac{1}{2}\%$ for collecting a note, his fee amounted to \$10.14. What was the amount of the note?

19. \$3367 was received to be invested after allowing a commission of 4% on amount invested. Find investment.

SOLUTION.

For each dollar invested the agent receives the dollar to be invested and \$.04 as commission, or \$1.04; hence the agent will invest as many dollars as \$1.04 is contained times in \$3367, or $\$3367 \div \$1.04 = \$3237.50$.

20. A commission merchant receives \$525 to buy goods on a commission of 5%. What was the amount invested?

21. A grain-buyer receives \$756 to buy wheat. If the commission is 3% and he charges 2% for guaranty of quality, how many bushels of wheat could be bought at 48¢ a bushel?

22. A note of \$3500 was sent to a collector to be collected. He secured 75% of the face of the note and charged 4% commission. How much should the collector remit?

23. The net proceeds of a shipment were \$501.80. If the rate of commission was 5%, and the other charges \$15, what was the gross amount of the sales?

SOLUTION.

$\$501.80 + \$15 = \$516.80 =$ amt. of sales less commission. On each dollar of sales the net proceeds would be $(\$1 - \$.05) = \$.95$; then the total amount of sales would be as many dollars as \$.95 is contained times in \$516.80. $\$516.80 \div .95 = \$544 =$ total sales.

24. If the net proceeds are \$620.75, and the rate of commission 3%, and guaranty $1\frac{1}{2}\%$, what were the gross sales?

25. Find the gross proceeds of a sale made by an agent charging 3% commission, $2\frac{1}{2}\%$ guaranty, \$14.80 for storage and \$10.50 for insurance, if the amount remitted as net proceeds was \$6117.20.

26. If an agent's commission was \$86.40 for buying 2560 yds. of cloth, at $\$1.12\frac{1}{2}$ per yard, what was the per cent rate of commission?

27. A commission merchant sold 14250 bu. of wheat at 52¢ per bushel for cash, and 5340 bu. at 53¢ per bushel, payable in 60 days. The rate of commission was 3%, for guaranty of time sales $1\frac{1}{2}\%$ and for storage \$26. What amount should he remit to his principal?

28. An agent sold 45 carriages at \$80 each, and 45 wagons at \$65 each, and remitted as net proceeds \$6068.25. Find the rate of commission.

29. What is the brokerage for selling 10500 bu. of corn at 38¢ per bushel, if the rate of brokerage is $\frac{1}{8}\%$?

30. I received from Milwaukee, 14000 bu. of wheat which I sold at 52¢ per bushel, on a commission of 3%, and invested the net proceeds in a grocery stock, for which I charged 6% commission. What was the total commission and how much was invested in groceries?

31. A commission merchant received a consignment of 360 bbl. of flour and advanced \$150 cash, paid \$38 freight, \$10.80 storage and \$9.50 drayage. He sold 175 bbl. at \$4.50, and 80 bbl. at \$4.25 and the remainder at \$4 per barrel. If the rate of commission for selling was $2\frac{1}{2}\%$, what was still due the consignor?

32. A grain-buyer purchased 6500 bu. of wheat at 55¢ per bushel, charging $\frac{3}{4}\%$ commission for buying, \$95 for drayage and \$135 for other expenses. What was the total cost?

33. An agent sent in a bill for \$3718.70 on account of purchases made, including his commission. If the rate of commission was $2\frac{1}{2}\%$, what was the value of the purchases?

34. A commission merchant sold 125 bbl. of flour at \$3.25 per barrel, and charged $2\frac{1}{2}\%$ for selling, $2\frac{1}{2}\%$ for guaranty, and \$25 for drayage. How many pounds of tea at 25¢ per pound would the net proceeds buy, if he charged 4% commission for buying?

35. The total cost of a purchase is \$1332.75. If the rate of commission is $5\frac{1}{2}\%$, and the other expenses \$14, what was the net cost?

36. I remitted \$1400 to an agent to invest in flour, after deducting all expenses. If flour is worth \$4 a barrel and his charges were 2% commission for buying, 2¢ per barrel for drayage, 10¢ per barrel for freight, and \$4.50 for advertising, how many barrels of flour would he buy and what balance remained in his hands?

SUGGESTION.—Deduct all charges, of which the gross amount is given, from amount of remittance; then find commission and other charges per barrel, which add to net cost of each barrel. Only whole barrels are considered; any balance is returned or placed to credit of principal.

STOCKS AND BONDS.

393. **Stock, or Capital Stock** is the name generally given to the capital of corporations, and is divided into equal parts, usually of \$100 each, called shares, the owners of which are called *stockholders*.

394. A **Corporation** is a number of persons who are authorized by law, or by charter, to transact business as a single individual.

395. A **Stock Certificate** is the paper issued by a corporation to its stockholders, certifying that they are the owners of a stated number of shares of its capital stock.

396. **Dividends** are so much of the net profits of the company as is divided among the stockholders.

397. An **Assessment** is a sum levied upon the stockholders to make up the losses, or to meet the expenses of the corporation. Dividends and

assessments are usually declared at a certain per cent of the value of the stock.

398. The **Gross Earnings** of a corporation include the total receipts from its business.

399. The **Net Earnings** of a corporation are the amount of the gross earnings left, after deducting all expenses, losses, and the interest upon its debts.

400. The **Par Value** of a share is the value specified in the certificate, and is usually \$100.

401. The **Market Value** of a share is the sum for which it can be sold. When a share is sold for more than its par value, it is said to be above par, or at a premium. Thus, stock sold at 105, means that each share is sold for \$105 or at a premium of 5%. When a share is sold for less than its par value it is said to be below par, or at a discount. Thus, stock sold at 98, means that each share is sold for \$98, or at a discount of 2%.

402. **Preferred Stock** is such stock that has preference in the declaring of dividends. This class of stock is entitled to a dividend at a certain rate per cent before any dividends can be declared on the common stock. Preferred stock is usually issued by a corporation as security for money borrowed.

403. **Watered Stock** is stock which is issued and for which payment is not required. This is very often done, when, by its charter, a corporation is not allowed to declare a dividend exceeding a certain rate per cent of the par value of its stock, or when the stockholders do not wish the public to know of the large dividends they are realizing. By issuing these additional shares to the stockholders, a lower rate of dividend can be declared, which will produce as much income as a higher rate on the original capital stock.

404. **Bonds** are written promises, made under seal, to pay a certain sum of money at a specified time. They are issued for money borrowed by the General Government, or by states, counties, cities, and business corporations, and bear interest. When bonds are issued by the Government and are payable at a certain time, the rate of interest with the due date, form the name by which they are generally known. As, U. S. 5's of 1881, Pacific 6's of 1884, etc.

405. Coupons are certificates of interest attached to bonds, representing the several instalments of interest. When a payment of interest becomes due, the coupon is detached from the bond and presented for payment.

406. Consols are English Government Bonds or Securities.

407. Rentes are French Government Securities.

408. Quotations are usually given at the price at which \$100 par value of the stocks or bonds can be bought or sold. This is not the case in all cities, as the par value of a share may be \$25, and be quoted at 23 or 28, while, in other cities, the same valuation may be quoted at 92 or 112. All quotations here given will be made on the basis of \$100 par value, unless otherwise stated.

When stocks or bonds are bought through a broker, the broker's brokerage charge, which is usually $\frac{1}{8}\%$, is added to the market value of the stock, or when sold through a broker, is deducted from the selling-price of the stock; thus, stocks bought at 98, with brokerage charge of $\frac{1}{8}\%$, would cost the purchaser $98\frac{1}{8}$; or if sold at 98, brokerage $\frac{1}{8}\%$, the seller would receive $97\frac{7}{8}$.

409. Computations are best made by considering all percentages based on the par value, as *so much money* per share, or bond, of \$100.

ILLUSTRATIVE EXAMPLES.

1. What cost 50 shares of Central Pacific at $93\frac{5}{8}$, brokerage $\frac{1}{8}$?

SOLUTION.

Quotation = \$93.625 per share.

Brokerage = \$.125 per share.

$\$93.625 + \$.125 = \$93.75$, total cost per share.

$\$93.75 \times 50 = \4687.50 . Ans.

2. Bought 20 shares of C. & A. R. R. at $102\frac{1}{4}$, brok. $\frac{1}{8}$, and sold at $98\frac{5}{8}$, brok. $\frac{1}{4}$. What sum did I lose?

SOLUTION.

$\$102.25 + \$.125 = \$102.375$, cost of each share.

$\$98.625 - \$.25 = \$98.375$, rec'd for each share.

$\$102.375 - \$98.375 = \$4$, loss on each share.

$\$4 \times 20 = \80 , total loss. Ans.

3. At what quotation must 5% bonds be purchased to yield an investment of 6%?

SOLUTION.

Each \$100 bond yields \$5 per annum.

$\$5 \div 6\% = \$83\frac{1}{3}$. Hence the required quotation is $83\frac{1}{3}$.

NOTE.—Quotations never involve other fractions than *halves*, *fourths* and *eighths*. In the last example the nearest quotation would be $83\frac{1}{3}$.

EXAMPLES FOR DRILL.

1. Bought 40 shares of stock at a premium of $4\frac{7}{8}\%$. Find cost. If the stock paid a dividend of 4%, find the dividend. Afterwards sold 25 shares at a discount of $9\frac{7}{8}\%$; what did I receive, brokerage being $\frac{1}{8}\%$ in each case?

SOLUTION.

Stock bought at a premium of $4\frac{7}{8}\%$ cost $104\frac{7}{8}$; then one share cost $\$104\frac{7}{8} + \frac{1}{8}\%$ brokerage = \$105. 40 shares cost $\$105 \times 40$, or \$4200 = cost. Par value of 40 shares is \$4000. $\$4000 \times .04 = \160 = dividend. If the stock sold at a discount of $9\frac{7}{8}\%$, selling-price would be $\$90\frac{1}{8}$ for \$100 share; $\$90\frac{1}{8} - \frac{1}{8}\%$ brokerage = \$90, net proceeds of one share; then $\$90 \times 25 = \2250 , net proceeds of 25 shares.

2. A speculator sold 135 shares of stock at $20\frac{1}{2}\%$ premium, paying $\frac{1}{8}\%$ brokerage. Find the net proceeds of the sale.

3. I bought 96 shares of stock at $14\frac{3}{4}\%$ discount, paying $\frac{1}{8}\%$ brokerage. Find total cost.

4. A man bought 90 shares of railway stock at $98\frac{1}{2}$, and sold the same at $5\frac{3}{8}\%$ premium. Brokerage being $\frac{1}{8}\%$ in each case, what was the net gain?

5. If 125 shares of stock were sold for \$13187.50, what was the selling-price per share and rate per cent of premium?

6. A person bought 55 shares of stock, paying therefore \$5073.75. Allowing $\frac{1}{8}\%$ for brokerage, at what rate were they bought?

7. I purchased bonds at 120, bearing interest at 6%, and also purchased bonds at 108, bearing interest at 5%. Which is the better investment?

8. An investor sold \$15000 of bonds, bearing interest at 5%, at 105, and invested the proceeds in bonds at 90, bearing interest at $4\frac{1}{2}\%$. What would be the difference in his income?

9. A gentleman purchased U. S. 5% bonds at $124\frac{7}{8}$, brokerage $\frac{1}{8}\%$ additional, and the semi-annual income produced by the investments was \$800. What amount did he invest?

10. A mining company divided \$48000 of its earnings among its stockholders. What was the rate per cent of dividend, the capital stock being \$800000? How much would a stockholder receive who owned 75 shares, of which the par value was \$50?

11. A man bought stock at $17\frac{5}{8}\%$ discount, and sold the same at $11\frac{3}{8}\%$ discount, making a net profit of \$540, brokerage being $\frac{1}{8}\%$ in each case. How many shares did he purchase?

TAXES.

410. A Tax is a sum of money levied for public purposes on the person, property or income of an individual.

411. A Property Tax is a tax levied upon property.

Property is of two kinds: **Real**, and **Personal**.

Real Property, or **Real Estate**, is immovable property, as buildings, lands, and their permanent improvements.

Personal Property is movable property, as money, merchandise, live stock, furniture, mortgages, stocks, etc.

412. A Poll Tax is a tax on each person not exempt by law.

413. Assessors are persons who appraise the property to be taxed and prepare the assessment roll.

An Assessment Roll is a list or schedule containing the names of persons taxed, the valuation of the property, the amount of their taxes, and the rate of taxation.

414. A Collector of Taxes is the person who has been appointed or elected to collect the taxes of a city, village, township, or any municipality.

ILLUSTRATIVE EXAMPLE.

The rate of taxation in a city is 30¢ on \$100 for state taxes; 34¢ for county taxes; and \$1.19 for school taxes. What tax would be paid by a person whose property is assessed at \$3200?

SOLUTION.

$\$.30 + \$.34 + \$1.19 = \1.83 , the total tax on \$100. $\$1.83 \div \$100 = \$.0183$, tax on \$1; therefore $\$.0183 \times 3200 = \58.56 , the required tax.

415. In computing property taxes the **Base** is the assessed valuation, and the **Percentage** is the amount of tax levied.

416. Tax-rates are usually expressed as so many cents or mills to the dollar of assessment; thus, should the tax-rate be .0075 of the assessment, the rate would be expressed "7½ mills to the dollar;" a rate of 1¼% would be expressed "1¢ 2½ mills to the dollar," or "12½ mills," etc. Sometimes the rate is expressed as so many dollars and cents on the \$100; thus, a rate of 3¾% might be called "a rate of \$3.75." In making computations, use the tax on each \$1 of assessment as the basis of calculation.

NOTE.—The *assessed value* of property is usually less than the true or *market value*.

EXAMPLES FOR DRILL.

1. What is the tax, if the assessed valuation of the property is \$6400, and the rate of taxation is \$1.17 on the \$100?
2. What is the tax, if the assessed valuation of the property is \$3400, and the rate of taxation is ½%?
3. The rate of taxation is 17 mills on the dollar. What would be the tax on property assessed at \$9000?
4. A taxpayer has real estate assessed at \$5000, and personal property assessed at \$3500, in a city in which the rate of taxation is \$1.20 on the \$100. Find the amount of his taxes.
5. What is the rate of taxation on \$100 if the assessed valuation of property is \$3000, and the total tax is \$37.50?

SOLUTION.

$\$37.50 \div \$3000 = \$.0125$, rate of taxation on \$1.
 $\$.0125 \times \$100 = \$1.25$, rate of taxation on \$100.

6. Find the rate of taxation on \$100 if the assessed valuation is \$3600, and the tax is \$42.12.

7. Find the rate per cent of taxation if the valuation of the property is \$13000, and the tax is \$65.

8. On property assessed at \$36400 the total property tax was \$546. Find the rate of taxation in mills on \$1.

9. A's property is assessed $\frac{2}{3}$ of its value ; the tax rate is $5\frac{1}{2}$ mills to the dollar, and his taxes amount to \$27.50. What is the value of his property?

10. The total assessed valuation of property in a city was \$13650-000, and the amount to be raised by taxation was \$150150. What would be the rate on \$100 to raise this amount of taxes?

11. The total assessed valuation of property is \$86000, the net tax is \$817, and the collector's commission is 5%. Find the rate of taxation on each \$100.

SOLUTION.

Since the collector receives \$.05 on every \$1 of tax collected, therefore \$.95 net tax remains on every \$1 of the gross tax; then $\$817 \div \$.95 = \$860$ gross tax, and $\$860 \div \$86000 = \$.01$ or \$1 on \$100.

12. What is the rate of taxation in a town whose taxable property is \$850000, the net tax being \$16150, and the collector's commission 5%?

13. A taxpayer has been assessed as follows: real estate \$7000; personal property \$2500; income \$4000. The income tax was 2%, and the rate of property tax is 17 mills on the dollar. What is the total tax if he pays in addition three polls of \$1.25 each?

14. The assessed valuation of the real estate of a county is \$2350-640, and the personal property \$876500; the year's expenses are: for schools \$10000, interest \$2579.96, roads \$8500, salaries \$6400, other expenses \$15000. If the county raises in addition to the above amount \$8500 to provide for a sinking-fund, and receives \$5800 from licenses, what tax must be levied on \$1 to meet the expenses?

15. In a certain town the assessment is \$875000, and the amount of taxes to be raised, \$10750. Compute the rate on the \$1 to tenths of a mill, and find Mr. Jones' tax, his assessment being \$17560. Also find Smith's assessment, his tax being \$82.40.

DUTIES, OR CUSTOMS.

417. Duties, or Customs, are taxes levied by the Government on imported goods.

418. An *Ad Valorem Duty* is a duty levied at a certain per cent of the cost of the goods in the country from which they are imported, as shown by the invoice.

NOTE.—Invoices of imports are made out in the weights, measures, and currency of the country from which the goods are imported.

419. *Specific Duties* are fixed taxes according to number, measure, or weight, levied upon certain specified articles.

NOTE.—In computing specific duty, 2240 lb. is considered a ton, and 112 lb. a hundredweight. Some goods are subject to both a specific and an *ad valorem* duty; as, cigars, \$2.25 per pound specific, and 30% *ad valorem*.

420. A *Custom House* is an office established by the Government for the transaction of business pertaining to duties. *Tariff* is a list or schedule of goods, with rates of duties imposed by law on the same. *Free List* is a list of goods that are exempt from duty. *Tare* is an allowance made for the box, crate, or bag containing the goods. *Gross Weight* is the total weight, before any allowance is made. *Net Weight* is the weight, after deducting all allowances. A *Bonded Warehouse* is a warehouse used for storing goods on which the duties have not been paid. A *Drawback* is the money refunded on goods, on which the import duties have been paid, and the goods are afterward exported.

ILLUSTRATIVE EXAMPLE.

What is the duty on 1000 yd. of Brussels carpet, 27 in. wide, invoiced at \$1.50 per yard, the rate of duty being 25¢ per square yard, and 35% *ad valorem*?

SOLUTION.

1000 yd. \times \$1.50 = \$1500, cost of the carpet.

\$1500 \times .35 = \$525, *ad valorem* duty.

27 in. = $\frac{3}{4}$ yd. width. $\frac{3}{4}$ yd. \times 1000 = 750 sq. yd.

750 yds. \times \$.25 = \$187.50, specific duty.

\$525 + \$187.50 = \$712.50, total duty. Ans.

EXAMPLES FOR DRILL.

NOTE.—For values of foreign money standards see table, p. 98.

Find the ad valorem duty on goods invoiced at—

1. £334 10s., rate of duty being 20%.
2. 900 francs, if the rate of duty is 60%.
3. 956.20 marks, allowing 5% for breakage, the rate of duty being 20%.
4. 3420 lire, allowing 10% for breakage, if the rate of duty is 40%.
5. £750 15s., allowing 3% for tare, if the rate of duty is 60%.
6. The invoice is £175 18s., and the rate of duty, 35%. Find the gross cost.
7. What is the specific duty on 750 boxes of cigars, 50 cigars in each, invoiced at \$65 per thousand, and weighing 10 lbs. per thousand, if the duty is \$2.25 per pound?
8. At 2¢ per dozen, what would be the duty on 1000 doz. eggs, allowing 10% for breakage?
9. What would be the duty, at \$2.25 per dozen, on 100 doz. bottles of wine, if an allowance of 5% is made for breakage?
10. Find the duty that must be paid for importing a block of marble, 12 ft. long, 4 ft. wide, and 2 ft. high, at 65¢ per cubic foot.
11. What is the duty on an importation of 4500 bu. of barley, valued at 52¢ per bushel, the rate of duty being 5¢ per bushel and 15% ad valorem?
12. An importation of goods from England was invoiced at £560 10s. If the goods weighed 340 lb., what would be the duty at 35¢ per pound specific, and 40% ad valorem?
13. An invoice of watches was imported from Switzerland, valued at 19340 francs. Find the duty at 45% ad valorem.
14. A manufacturer of woolens imported 125700 lb. of wool, invoiced at 1s. a pound which was subject to a specific duty of 10¢ a pound, and 35% ad valorem. What was the total duty? What was the gross cost of the wool per pound?

INSURANCE.

421. Insurance is a contract by which one party agrees to indemnify another for loss or damage within a given time. It is of two kinds: Property Insurance and Personal Insurance.

422. Property Insurance includes:—

- I. Fire Insurance, which is indemnity for loss of property by fire.
- II. Marine Insurance, which is indemnity for loss or damage to a ship or its cargo.
- III. Live Stock Insurance, which is indemnity for loss of horses, cattle, etc., from lightning or other casualties.

423. Personal Insurance includes:—

- I. Life Insurance, which is indemnity for loss of life.
- II. Accident Insurance, which is indemnity for loss occasioned by accidents.
- III. Health Insurance, which secures a weekly allowance in case of sickness.

NOTE.—*Transit Insurance* is indemnity against loss to property by transportation.

424. Insurers, or Underwriters, are the parties or companies issuing the insurance. Insuring anything is often called “taking a risk.”

425. The Premium is the price paid for the insurance, and is usually estimated at a certain per cent of the amount insured, or at a certain sum for \$100. A rate of \$.90 means \$.90 on \$100. Rates vary according to the risk involved.

426. Short Rates are rates for less than a year and are considerably higher, proportionately, than long-time rates.

427. To Cancel a policy is to annul the contract between the insurance company and the insured parties.

When an insurance policy is canceled at the request of the holder, the company has the right to retain the customary short rates for the time the policy has been in force. But, where it is canceled at the request of the company, a pro rata of the premium paid is returned for the unexpired time of the policy.

428. The Term of Insurance is the time during which the policy is to be in force.

429. The Policy is the contract of insurance between the person whose property is insured and the insurance company. It contains a description of the property insured, the amount of the insurance, and the conditions under which the risk is taken. **A Valued, or Closed Policy** is the ordinary form, and contains a certain fixed valuation on the thing insured. **An Open Policy** is one in which the value of the article insured is to be determined in case of loss.

NOTE 1.—Fire insurance companies do not usually insure property for its full value, in order that the person, whose property is insured, may have an interest in its protection.

NOTE 2.—Fire insurance companies are also liable for loss or damage caused by water, if used for the purpose of extinguishing a fire, also for damage or loss of buildings torn down or otherwise damaged to prevent a fire from spreading. They also reserve the right to rebuild or repair damaged property.

NOTE 3.—Ordinarily if the loss is partial and less than the amount of the insurance policy, the company will pay the full amount of loss, and if insured in two or more companies, each company will pay its pro rata share of the loss.

430. Insurance Companies are of two kinds:—

I. **Stock Insurance Companies** in which the paid-up capital is owned by the stockholders, and they alone share the profits, and are liable for the losses.

II. **Mutual Companies** in which the losses are shared by the parties insured.

NOTE.—Some companies combine the principles of the stock and mutual companies and are called *Mixed Companies*.

In these companies a limited dividend is first paid to the stockholders, and the profit, over and above this dividend is divided among the participating policy-holders.

These companies also issue non-participating policies, the holders of which do not share the gains or losses of the company.

EXAMPLES FOR DRILL.

1. • A house was insured for \$3500 at a premium of $1\frac{1}{2}\%$. What was the cost of the insurance?

SOLUTION.—Amount of insurance (\$3500) \times rate of premium ($1\frac{1}{2}\%$) = \$52.50, cost of insurance.

2. An insurance company took a risk on a store and its contents valued at \$35000, for $\frac{3}{4}$ of its value, at a premium of \$.90 on \$100. What was the amount of premium paid?

3. A factory is insured for \$8000 in the Phoenix Fire Insurance Co., and in the Hartford Insurance Co. for \$12000. If a fire damaged the property to the extent of \$8000, what portion of the damage should each company pay?

SOLUTION.

Total insurance = \$20000; then $\$8000 \div \$20000 = .40$, or 40%. Therefore each company will have to pay 40% of the insurance held by them.

$\$8000 \times .40 = \3200 to be paid by the Phoenix Ins. Co.

$\$12000 \times .40 = \4800 to be paid by the Hartford Ins. Co.

4. A warehouse and its contents valued at \$40000, were insured as follows: \$5000 on warehouse, and \$10000 on contents, in the Home Insurance Co.; \$2500 on warehouse, and \$7500 on contents, in the Rochester German Insurance Co. If the contents were damaged by fire to the extent of \$15000 and the warehouse to the extent of \$5000, what portion of the loss should be paid by each company?

5. A store and contents were insured as follows: \$8000 in the Queen Insurance Co., \$6000 in the Mechanics' Insurance Co., and \$4000 in the Globe Fire Insurance Co. A loss of \$4000 having been sustained, how much should each company contribute?

6. A block of buildings was insured for three years for \$15000 at $1\frac{1}{2}\%$. At the end of the first year, the policy was canceled at the request of the owner of the buildings. What was the return premium, if the short rate for one year was \$.70 per \$100.

7. I paid a premium of \$54 for insuring a house for \$6000. What was the rate of insurance in cents on \$100?

SOLUTION.

Total premium (\$54) \div total insurance (\$6000) = rate of premium on \$1; this rate multiplied by 100 = rate on \$100, or 90¢.

8. An insurance company receives a premium of \$187.50 on property insured for \$15000. What was the rate per cent of premium?

9. I insure my dry goods store, valued at \$18000, and the contents valued at \$24000, for $\frac{2}{3}$ of their value, paying as premium \$350. At the

same rate what should I pay for insuring a brick block valued at \$35750 for $\frac{3}{4}$ of its value?

10. For what amount must a consignment of goods valued at \$4850 be insured at a premium of 3%, so that, in case of total loss, the value of the goods and premium may be recovered?

SUGGESTION.—Since the face of the policy is 100% and the premium is 3%, the value of the goods, \$4850, = 97% of the policy, or amount of insurance.

11. A cargo of grain valued at \$20000 was insured at 2% to cover $\frac{3}{4}$ of its value and the premium. What was the amount of the insurance?

12. For what amount must a house valued at \$7000 be insured at \$1.25 per \$100 to cover $\frac{4}{5}$ of its value and the premium?

13. A consignment of 7000 bu. of wheat, worth 55¢ per bushel, is insured for $\frac{9}{10}$ of its value at $1\frac{1}{2}\%$ premium. Should the wheat be totally destroyed what would be the amount of the loss?

14. The Mutual Insurance Co. insured a block for \$45000, at a premium of \$.75 per \$100; the Mutual then reinsured \$10000 in the Home at a premium of 80¢, and \$10000 in the Etna at a premium of $\frac{5}{8}\%$. How much premium did each company receive? What was the gain or loss of the Mutual?

NOTE.—There is little uniformity in the methods of making computations in Life Insurance, different companies following different systems. For this reason problems involving life insurance are omitted.

EQUATION OF ACCOUNTS.

431. The Equation of Accounts is the process of adjusting the mutual accounts of Debtor and Creditor upon an equitable basis. The subject includes such business applications as Equation of Payments, Averaging Accounts, Cash Balances, Adjustment of Accounts, Current, and Account Sales, etc.

432. The methods of adjusting accounts rest upon the theory that

- I. On all debts settled before maturity, discounts should be allowed, and
- II. On all debts settled after maturity, interest shou'd be charged.

EQUATION OF PAYMENTS.

433. Equation of Payments is the process of finding the time when two or more debts due at different dates can be paid in one payment without loss to either the debtor or creditor.

434. The Average, or Equated Date is the date on which two or more debts can be equitably discharged by one payment.

435. The Focal Date is any assumed date with which the dates of several amounts are compared to find the average date.

ILLUSTRATIVE EXAMPLE.

An invoice of goods amounting to \$1500, was bought on the following terms: \$400 on 2 months' credit; \$500 on 5 months' credit; \$600 on 7 months' credit. What time should be allowed for the payment of the whole at one time?

SOLUTION.

$$\begin{array}{rcl}
 400 \times 2 \text{ mo.} & = & \$ 800 \text{ for 1 mo.} \\
 500 \times 5 \text{ mo.} & = & \$2500 \text{ " " " } \\
 600 \times 7 \text{ mo.} & = & \$4200 \text{ " " " } \\
 \hline
 1500 & & \$7500 \text{ " " " } \\
 7500 \div 1500 & = & 5 \text{ months, Equated Time.}
 \end{array}$$

EXPLANATION.—If the first payment should be made immediately, the buyer would lose the use of \$400 for 2 months, equal to \$800 for one month; if the second payment was made immediately, the

buyer would lose the use of \$500 for 5 months, equal to \$2500 for 1 month; if the third payment was made immediately, the buyer would lose the use of \$600 for 7 months, equal to a loss of \$4200 for 1 month; then the total loss by making these payments immediately would be equal to a loss of $\$800 + \$2500 + \$4200 = \7500 for 1 month. To avoid this loss, the buyer must keep the \$1500 as many months as \$1500 is contained times in \$7500, or 5 months.

EXAMPLES FOR DRILL.

1. A merchant bought a bill of goods amounting to \$2000 on the following terms: \$500 payable in 2 months; \$700 payable in 4 months; and the balance in 6 months. What is the equated date for the payment of the entire bill?

2. On July 18, 1894, a merchant bought an invoice of goods amounting to \$2400, and agreed to pay $\frac{1}{2}$ in 60 days and the remainder in 90 days. He afterwards wishes to pay for the invoice in one amount. On what date should this payment be made?

3. On May 14, 1894, the following invoices of goods were bought: \$300 on 4 months' credit; \$500 on 6 months' credit; \$800 on 2 months' credit; \$600 on 5 months' credit. Find the equated time of payment.

4. A wholesale merchant sold the following invoices on Jan. 15, 1895: \$225 on 30 days' credit; \$900 on 90 days' credit; \$650 on 60 days' credit; \$340 on 30 days' credit. On what date could payment be made of the whole indebtedness without loss to either party?

436. To find the equated time for the payment of two or more debts contracted at different dates.

There are two current methods of solving problems involving the principles underlying the Equation of Accounts. These are known respectively as "The Product Method," and "The Interest Method." The former of these is the more direct, and involves less calculation, but the latter is, perhaps, more generally used by accountants. The student should become familiar with both methods. We give a full exposition of each.

ILLUSTRATIVE EXAMPLE.

J. L. Perkins bought of Campbell & Co. the following bills: Mar. 20, 1894, \$400; April 19, 1894, \$600; June 30, 1894, \$500. What was the equated time of payment?

INTEREST METHOD.

Assume June 30, 1894, as the focal date, and 6% as the rate of interest.

FIRST SOLUTION.

Mar. 20, 1894, \$400, interest for 102 days =	\$ 6.80
Apr. 19, 1894, \$600, interest for 72 days =	\$ 7.20
June 30, 1894, \$500, interest for 000 days =	\$ 0.00
<u>\$1500</u>	<u>\$14.00.</u>

Interest on \$1500 for 1 day = \$.25.

$\$14 \div \$.25 = 56$ days.

Then the equated date will be 56 days prior to June 30, 1894, or May 5, 1894.

EXPLANATION.—Assume June 30 as the focal date; then if the amount (\$400) due on March 20 is not paid until June 30, interest should be paid on it for that time (102 days), or \$6.80 interest; if the amount (\$600) due April 19 is not paid until June 30, interest should be paid on it for that

time (72 days), or \$7.20 interest; if the amount (\$500) due June 30 is paid on that date no interest will be required; therefore the total amount due June 30 would be \$1500, the amount of the debt, and also \$14 interest, or the cash balance due June 30 is \$1514; but when is the face of the debt (\$1500) due? If by settlement on June 30, \$14 interest would have to be paid, therefore, in order that no interest should be paid, the date of settlement should be as many days prior to June 30 as \$14 is days' interest on \$1500; the interest on \$1500 for 1 day at 6% is \$.25; then \$14 is the interest for as many days as \$.25 is contained times in \$14, or 56 days, and 56 days prior to June 30 is May 5, the equated date.

RULE.—*Select the latest date as the focal date, and compute the interest on each item from its date to the focal date; divide the interest on the items by the interest on the face for one day and the result will be the number of days average time. Count back from the focal date the number of days thus found, this date will be the due date of the face of the account.*

NOTE.—Some accountants use 1%, or 12%, as an interest rate. The result is, of course, the same, regardless of the rate taken. Because of simplicity of calculation, 6% is here recommended.

PRODUCT METHOD.

Assume as focal date June 30.

SECOND SOLUTION.

Mar. 20, \$400 × 102 (time from March 20 to June 30)	= \$40800.
Apr. 19, \$600 × 72 (time from April 19 to June 30)	= \$43200.
June 30, \$500 × 000	= \$00000.
<u>\$1500 × ?</u>	<u>= \$84000.</u>

$\$84000 \div \$1500 = 56$ days, and 56 days prior to June 30 is May 5, 1894.

RULES.

I. *Assume as a focal date the latest date on which any item of the account falls due. Multiply each item by the difference in days between its due date and the focal date.*

II. *Divide the sum of the products thus obtained by the sum of the debts to find the number of days to count backward from the focal date.*

NOTE 1.—In finding the average time in days, fractions of a day of one-half or more are counted as a full day, fractions of a day of less than one-half are dropped.

NOTE 2.—Mathematically speaking, any focal date may be chosen, but one multiplication is saved by selecting the latest date upon which any item in the account falls due. Some accountants select the *earliest* date on which any item falls due, and then add the equated time to this date.

EXAMPLES FOR DRILL.

1. A merchant bought the following merchandise : Feb. 25, 1894, a bill amounting to \$400; Mar. 15, 1894, a bill amounting to \$500; Apr. 5, 1894, a bill amounting to \$250; Apr. 15, 1894, a bill amounting to \$450. Find the equated date of purchase.

2. A commission merchant made the following sales : Oct. 15, 1893, 50 bbl. of apples at \$3.25 per barrel; Oct. 25, 30 bbl. of apples at \$3.50 per barrel; Nov. 5, 100 bbl. of apples at \$3.30 per barrel; Nov. 15, 40 bbl. of apples at \$3.50 per barrel. What is the equated date of the sales ?

3. Brownell bought of M. M. Bernard, July 15, 1894, merchandise amounting to \$350; Aug. 20, 1894, merchandise amounting to \$400; Aug. 30, 1894, merchandise amounting to \$200; Sept. 1, 1894, merchandise amounting to \$450; Sept. 15, 1894, merchandise amounting to \$600. Find the equated date of purchase.

4. On Sept. 15, 1894, I sold an invoice of goods amounting to \$320; Oct. 1, 1894, an invoice amounting to \$530; Oct. 15, 1894, an invoice amounting to \$420; Nov. 30, 1894, \$560. Find the equated date of the sale.

5. Sold Gilroy & Co. goods as follows : Aug. 15, 1892, an invoice amounting to \$300 on 60 days' credit; Aug. 30, 1892, an invoice amounting to \$360 on 60 days' credit; Oct. 15, 1892, an invoice amounting to \$450 on 60 days' credit; Oct. 25, 1892, an invoice amounting to \$250 on 60 days' credit. Find the equated date of payment.

NOTE.—When the same credit is allowed on all invoices, find the equated date of payment and add to this date the number of days' credit allowed.

6. A wholesale merchant sold goods as follows : July 15, 1894, \$500 on 60 days' credit; July 30, 1894, \$400 on 90 days' credit; Aug. 15, 1894, \$300 on 30 days' credit; Aug. 25, 1894, \$320 cash. What is the equated time for paying the entire amount ?

NOTE.—First find the due date of each amount by adding the term of credit to the date of each purchase, then proceed to find the equated time as in the previous problems.

7. A jobber sold Feb. 1, 1893, 100 bbl. flour at \$4.25 per barrel on 1 month's credit; Feb. 15, 1893, 25 bbl. of apples at \$3.75 per barrel on 2 months' credit; Mar. 1, 1893, 325 lb. butter at \$.26½ per pound on 30 days' credit; Mar. 30, 1893, 150 bbl. flour at \$3.40 per bbl. on 60 days' credit. Find the average date of payment.

8. A merchant bought an invoice of goods amounting to \$3000 on July 16, 1894, on the following terms: \$1200 payable in 30 days; \$1000 in 60 days; and the balance in 90 days. Find the equated date of payment.

9. L. W. Winslow bought of L. M. King & Son: Feb. 16, 1894, per bill rendered \$50; Feb. 28, 1894, per bill rendered 1 month, \$120; Mar. 5, 1894, per bill rendered 30 days, \$400; Mar. 25, 1894, per bill rendered 1 month, \$250; Apr. 28, 1894, per bill rendered 2 months, \$240; July 26, 1894, per bill rendered 60 days, \$200. Find the equated date of payment.

10. Spencer & Co. bought of Field & Co.

1892.

Oct. 15,	To Merchandise,	4 months,	\$500.
Oct. 30,	"	" 4 "	350.
Dec. 15,	"	" 90 days,	400.
Dec. 30,	"	" 30 "	700.

1893.

Jan. 15,	To Merchandise,	2 months,	\$600.
Feb. 15,	"	" 60 days,	400.

Find the equated time of payment.

AVERAGING ACCOUNTS.

437. *Averaging Accounts* is the process of finding at what time the balance of an account can be paid without loss to either party.

438. If the balance of an account is paid before the average due date, it is customary to deduct bank discount on the balance of the account from the date of its payment to the average due date; if paid after, interest at the current rate is added to the balance of the account, from the average date to the date of settlement.

ILLUSTRATIVE EXAMPLE.

What is the equated time of payment of the balance of the following account?

Dr.		JAMES G. GRILL.		Cr.	
1893.			1893.		
Mar. 13	To Mdse.,	\$800	Mar. 31	By Cash,	\$400
Apr. 6	“ “	400	Apr. 30	“ “	200

Assume the latest date as the focal date. (Apr. 30, 1893.)

SOLUTION.

INTEREST METHOD.

Mar. 13, \$800 (Int. 48 da.) = \$6.40.	Mar. 31, \$400 (Int. 30 da.) = \$2.
Apr. 6, 400 (Int. 24 da.) = 1.60.	Apr. 30, 200 (Int. 00 da.) = 0.
<u>\$1200</u>	<u>\$8.00.</u>
600	2.00.
<u>\$ 600</u>	<u>\$6.00.</u>

Interest on \$600 for 1 day = \$.10.

\$6 ÷ \$.10 = 60 days prior to April 30, or March 1, equated date.

EXPLANATION.—Assume the latest date (Apr. 30) as focal date. Find the interest on each debit item from given date to the focal date; also find the interest on each credit item from given date to the focal date. The total debit interest is \$8, and the total credit interest is \$2; hence the amount due Apr. 30, is \$600 balance of account and \$6 interest. Since the balance of the account (\$600) has been owing long enough to accumulate \$6 interest, and as \$6 is the interest on \$600 for 60 days, the equated date of payment is 60 days prior to Apr. 30, or Mar. 1.

RULES.

I. Add the term of credit to the date of purchase of such items as are sold on credit.

II. Assume the latest date on which any item falls due or any payment is made, as focal date, and find the interest on each item, both debit and credit, from the date of its maturity, or the given date, to the focal date; then find the balance of interest and the balance of the account, and divide the balance of interest by the interest on the balance of the account for one day, and the result will be the time in days to count back or forward from the focal date.

NOTE.—When the balance of interest and the balance of the account are on the same side, count back from the focal date, if the latest date is selected, to find the average date; if the balance of the account and the balance of the interest are on different sides, count forward from the focal date to determine the average date. When the earliest date is selected as the focal date and the balance of the account and the balance of the interest are on the same side, count forward from the focal date; and when the balance of the account and the balance of the interest are on opposite sides, count backward from the focal date to find the average date.

Focal Date, April 30.

SOLUTION II.

Dr.

		For 1 day.
Mar. 13, To \$800	$\times 48 =$	38400.
Apr. 6, " 400	$\times 24 =$	9600.
	<hr/>	<hr/>
	1200	48000.
Deduct 600		12000.
	<hr/>	<hr/>
	600	36000.

PRODUCT METHOD.

Cr.

		For 1 day.
Mar. 31, By 400	$\times 30 =$	12000.
Apr. 30, " 200	$\times 0 =$	0
	<hr/>	<hr/>
	600	12000.

$$\$36000 \div 600 = 60 \text{ days.}$$

April 30 — 60 days = March 1, equated date. Ans.

EXPLANATION.—Had the account been settled Apr. 30, Grill should be charged with the use of \$800 for 48 days, and \$400 for 24 days = \$48000 for 1 day. He should be credited with the loss of the use of \$400 for 30 days and \$200 for 0 days = \$12000 for 1 day. He is therefore to be charged with \$48000 for 1 day — \$12000 for 1 day = \$36000 for 1 day. This is equal to the use of \$600 (the sum he still owes) for \$36000 \div 600, or 60 days. The equated time of payment is therefore, 60 days before Apr. 30, or Mar. 1.

RULES.

I. Add the term of credit of each item, to the date of such items as are sold on credit.

II. Assume as focal date the latest date on which any item falls due or any payment is made, and multiply each item of the account by the time in days between the focal date and the date on which each item falls due.

III. Find the sum of the products of each side of the account, and divide the difference between the sum of the debit and credit products by the balance of the account, and the quotient will denote the number of days between the focal date and the equated date, which should be counted back,

if the balance of the products and the balance of the account items are on the same side of the account; and counted forward from the focal date, if the product balance and the balance of the accounts are on opposite sides.

EXAMPLES FOR DRILL.

1. Find the equated date for paying the balance of the following account :—

<i>Dr.</i>			H. E. WEST.	<i>Cr.</i>		
<i>1892.</i>				<i>1892.</i>		
July 15	To Mdse.	\$350		Aug. 14	By Cash,	\$200

2. When is the balance of the following account due by equation?

<i>Dr.</i>			G. H. BLISS.	<i>Cr.</i>		
<i>1892.</i>				<i>1892.</i>		
Jan. 15	To Mdse.,	\$300		Jan. 30	By Cash,	\$125
Jan. 31	" "	250		Mar. 15	" "	220

3. Find the equated date of settlement of the following account :—

<i>Dr.</i>			GEO. A. GILSON.	<i>Cr.</i>		
<i>1893.</i>				<i>1893.</i>		
June 15	To Mdse.,	\$250		July 15	By Cash,	\$300
July 15	" "	400		Aug. 25	" "	220
Aug. 30	" "	650		Sept. 15	" "	125

4. When is the balance of the following account due by equation?

<i>Dr.</i>			HOFFMAN & Co.	<i>Cr.</i>		
<i>1894.</i>				<i>1894.</i>		
Mar. 15	To Mdse., 1 mo.	\$500		Apr. 15	By Cash,	\$125
Apr. 14	" " 1 "	340		May 15	" "	125
May 30	" " 2 "	300		June 15	" "	125
June 30	" "	350		July 15	" "	125

NOTE.—In the above problem, add the term of credit to the date of each item to find the due date, and proceed as before.

5. Find the equated date for the payment of the balance of the following account:—

<i>Dr.</i>			<i>J. W. ALLEN.</i>			<i>Cr.</i>		
<i>1893.</i>						<i>1893.</i>		
Sept. 1	To Mdse., 60 da.	\$340	Oct. 25	By Cash,			\$378	
Oct. 15	" " 60 "	524	Nov. 16	" "			140	
Nov. 15	" " 30 "	364	<i>1894.</i>					
Dec. 1	" " 90 "	325	Jan. 15	" "			640	

CASH BALANCES.

439. The Cash Balance of any account may be found by first ascertaining the equated date of the payments, and computing the interest on the balance due at that time, from the equated date to the specified date; or if the equated date is later than the date of settlement, discount the balance of the account for the intervening time.

NOTE.—The Cash Balance may also be found by computing the interest on each item from its due date to the specified date. The difference between the total debit interest and the total credit interest is the balance of interest; then if the balance of the account and the balance of interest are on the same side, their sum will be the cash balance due. If the balance of the account and the balance of interest are on opposite sides, their difference will be the cash balance due.

6. Find the cash balance of the following account, due Jan. 1, 1895, money being worth 6%.

NOTE.—Find time by compound subtraction when longer than 1 year.

<i>Dr.</i>			<i>N. S. PHELPS.</i>			<i>Cr.</i>		
<i>1894.</i>						<i>1894.</i>		
Mar. 15	To Mdse., 30 da.	\$500	June 15	By Cash,			\$350	
Apr. 25	" " 1 mo.	300	Sept. 14	" "			240	
July 25	" " 2 mo.	400	Dec. 24	" "			225	

7. Find the cash balance of the following account on Nov. 15, 1894, 6% Int.

AVERAGING ACCOUNT SALES

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Dr. J. R. REED in account with S. B. EMERY. *Cr.*

1893.			1893.		
Apr. 15	To Mdse., 2 mo.	\$640	June 1	By Cash,	\$250
May 25	" " 1 mo.	375	Aug. 1	" "	300
June 26	" "	500	Sept. 15	" "	400
Aug. 24	" "	320	Oct. 15	" "	200

8. The following account was settled Apr. 1, 1895. Find the amount paid, interest at 6%.

Dr. J. W. LEE in account with CHAS. R. BINDER. *Cr.*

1894.			1894.		
May 15	To Mdse., 1 mo.	\$345	June 15	By Cash,	\$150
June 28	" " 30 da.	725	July 15	" "	150
Aug. 28	" " 10 "	940	Sept. 15	" "	300
Sept. 5	" Cash,	400	Oct. 1	" "	200
Sept. 16	" Mdse., 2 mo.	500	Nov. 24	" "	100
Nov. 15	" "	400	Dec. 31	" "	200

9. The current interest rate being 10%, find the cash balance of the following account on Apr. 15, 1895.

Dr. R. S. MILLER in account with DAVIS, BROWN & Co. *Cr.*

1895.			1895.		
Jan. 2	To Mdse., 60 da.	\$ 560	Jan. 27	By Cash,	\$ 200
" 15	" " 90 "	970	Feb. 16	" Dft. 10 da.,	350
Feb. 3	" " 30 "	1250	Mar. 20	" " 30 "	675
Mar. 17	" " 2 mo.	520	Apr. 2	" Note 60 "	1000
Apr. 9	" " 3 "	800			

AVERAGING ACCOUNT SALES.

440. When goods are received to be sold on commission, the consignee usually pays the freight and other items of expense, and very often makes cash advances on the consignment, and charges interest on same, or uses the net proceeds for a sufficient time to offset the accrued interest on these advances.

ILLUSTRATIVE EXAMPLE.

Find the equated date for paying the net proceeds of the following account sales of 200 bbl. of apples.

1894.

Mar. 15, sold 75 bbl. at \$4 cash	\$300	
Apr. 14, sold 50 bbl. at \$5 cash	\$250	
May 14, sold 75 bbl. at \$4 cash	\$300	\$850

CHARGES.

Mar. 10, Cash for freight	\$ 90	
“ “ “ “ insurance and advertising	\$ 15	
Apr. 14, “ advanced	\$200	
May 14, Storage	\$ 10	
Commission at 4% (on \$850)	\$ 34	\$349
Net Proceeds		<u>\$501</u>

EQUATING OF SALES AND COMMISSION.

SOLUTION.

INTEREST METHOD.

Focal date May 14, 1894.

Mar. 15, \$300 int. 60 da. = \$3.00.	Int. on \$850 for 1 da. = \$.14 $\frac{1}{2}$.
Apr. 14, \$250 int. 30 da. = \$1.25.	\$4.25 \div \$.14 $\frac{1}{2}$ = 30 days.
May 14, \$300 int. 00 da. = 00.00.	30 days prior to May 14, is Apr. 14,
<u>\$850</u>	<u>\$4.25.</u> equated date of sales and commission.

EXPLANATION.—First find the equated date of the sales as in equation of payments. Take this date (Apr. 14), as the date of the total sales (\$850), and of the commission (\$34); then place the charges and their dates on the debit side, the total sales and their equated date on the credit side, and average the account.

EQUATING NET PROCEEDS.

Focal date May 14.

Mar. 10, \$90		
“ “ 15		
<u>\$105</u>	Int. 65 da. \$1.14	Apr. 14, \$850 Int. 30 da. \$4.25
Apr. 14, 200 “ 30 “ 1.00		<u>349</u> <u>2.31</u>
May 14, 10 “ 00 “ 0.00		<u>\$501</u> <u>\$1.94</u>
Apr. 14, <u>34</u> “ 30 “ .17	Int. on \$501 for 1 da. = \$.083 $\frac{1}{2}$	
<u>\$349</u>	<u>\$2.31</u>	<u>\$1.94 \div \$.083$\frac{1}{2}$ = 23 days.</u>

23 days prior to May 14 = April 21, the equated date for paying Net Proceeds.

EXAMPLE FOR DRILL.

1. When are the net proceeds of the following account sales due?

CHICAGO, ILL., June 1, 1894.

Account Sales of Flour,

Sold on account of H. R. POWERS.

1894.							
Mar.	1	80 bbl. to H. C. Henson @ \$12	***	**			
Apr.	15	50 " " J. F. Johnson @ 13	***	**			
May	1	60 " " L. M. Culver @ 12 ⁵⁰	****	**			
"	22	75 " " H. C. Wells @ 12	***	**	****	**	
CHARGES:							
Feb.	28	Freight.....	\$190	00			
"	28	Cartage	20	00			
"	28	Insurance.....	30	00	***		
Apr.	15	Cash advanced on Consignment. ...			\$500	**	
		Commission 3%	**	**			

BANKING AND EXCHANGE.

441. Exchange is a method of making payments by means of written orders, variously known as checks, drafts, or bills of exchange.

442. A Bank is a corporate institution established for the purpose of effecting exchanges, receiving deposits, emitting bills, or loaning money.

443. A Deposit is a sum of money deposited at a bank and which may be drawn out by the depositor, through his written orders, or *Checks*.

NOTE.—A *Special Deposit* is a sum of money or other valuable package placed in a bank, for safe keeping. In an ordinary deposit the *identical* money or other security is not returned to the depositor or his order but merely its equivalent in legal tender money. In the case of a special deposit the *identical package* must be returned.

444. A Draft is a written order given by one person to another, directing a specified sum of money to be paid to a third party at a specified time. Drafts are frequently called "bills."

NOTE.—Drafts are usually drawn by one bank upon another, and for the purpose of saving the transmission of money. Thus if A, in Chicago, wishes to send B, in New York, a sum of money, he deposits the money with a Chicago bank, receiving in exchange a draft on a New York bank with which the Chicago bank has dealings, and which he sends to B, instead of the money.

445. Domestic Exchange is the exchange between two cities of the same country.

446. Foreign Exchange is the exchange between cities in different countries. Drafts on foreign countries are usually called **Bills of Exchange**. They are written in "sets" of three.

447. Sight Drafts are those payable on presentation.

448. Time Drafts are those payable a certain number of days after "sight," or presentation.

449. Banks usually charge a certain per cent of the face of a draft for issuing it.

This charge is variously called "premium," "exchange," or "brokerage." Sometimes, however, in the course of trade the bills of one city accumulate in another, in which case the exchange may fall to a *discount*.

The Course, or Price of Exchange, is the market value of \$1 of exchange. Thus, if one pays \$1.0025 for a draft of \$1, the course of exchange is $1.00\frac{1}{4}$ and exchange is said to be " $\frac{1}{4}\%$ premium," or " $\frac{1}{4}\%$ above par."

NOTE.—In Foreign Exchange the term "Course of Exchange" implies the variation of commercial exchange from the standard coinage exchange in force between the countries. Thus, the Course of Exchange on London may be \$4.84 or \$4.89, instead of \$4.8665, the Standard rate.

Sometimes in large bills an additional charge of "brokerage" is made by the dealer in exchange. This charge is added to the regular course of exchange.

450. In issuing time drafts, it is customary to deduct from the cost of the draft the *bank discount* on the face of the draft for the time for which it is drawn plus *three days of grace*.

451. In making computations in exchange it is usually best to deal with the cost of \$1 of exchange as a basis. Though in some cases the operation may be shortened by finding the whole premium or discount, and subtracting it or adding to the face of the draft.

452. To find the Cost, or Cash Value, of Drafts.**ILLUSTRATIVE EXAMPLES.**

With exchange at $2\frac{1}{2}\%$ premium find the cash cost of a draft for \$500.

FIRST SOLUTION.

$$\$1.025 = \text{Cost of \$1 of exchange.}$$

$$\$1.025 \times 500 = \$512.50. \quad \text{Ans.}$$

SECOND SOLUTION.

$$\frac{1}{2}\% \text{ of } \$500 = \$12.50.$$

$$\$500 + \$12.50 = \$512.50. \quad \text{Ans.}$$

With exchange $\frac{3}{4}\%$ discount find the cost of a draft for \$300.

FIRST SOLUTION.

$$$.9925 = \text{Cost of \$1 exchange.}$$

$$.9925 \times 300 = \$297.75. \quad \text{Ans.}$$

SECOND SOLUTION.

$$\frac{3}{4}\% \text{ of } \$300 = \$2.25.$$

$$\$300 - \$2.25 = \$297.75. \quad \text{Ans.}$$

EXAMPLES FOR DRILL.

1. Find the value of a draft for \$275.15 when exchange is $\frac{1}{8}\%$ premium.

2. What cost a draft on London for £59 7s. with exchange 4.845 and brokerage $\frac{1}{2}\%$?

NOTE.—In buying a draft the brokerage is added, and in selling it is subtracted, from the course of exchange.

3. What cost a draft on New York for \$179.25 exchange $\frac{3}{4}\%$ below par?

4. Find the cost of a Paris bill for 1275 fr. at \$.187, brokerage $\frac{1}{4}\%$.

5. The premium on a draft on San Francisco at $\frac{1}{4}\%$ was \$2.75. What was the face of the draft?

6. What cost a 60-day draft on Chicago for \$700, exchange $\frac{1}{2}\%$ premium, and discount 12%?

SOLUTION.

\$1.005 course of exchange (sight).

\$.021 Bank discount for 63 da. on \$1.

\$1.005 — .021 = \$.984, cost of a draft for \$1.

.984 \times 700 = \$688.80, cost of draft. Ans.

7. With discount at 6% and exchange $\frac{1}{4}\%$ premium, what cost a 30-day draft on New York for \$750?

8. With exchange 4.85, brokerage $\frac{1}{4}\%$, and discount 6%, what cost a 60-day draft on Liverpool, Eng., for £1975 16s.?

9. Find cash value, or proceeds, of a 10-day draft on St. Louis, for \$1500, exchange $\frac{3}{4}\%$ discount, brokerage $\frac{1}{4}\%$ and discount 12%.

SUGGESTION.— Here, subtract the brokerage.

453. To find the Face of a Draft or Bill.

When exchange is $\frac{1}{2}\%$ premium, required the face of a sight-draft that can be bought for \$500.

SOLUTION.

\$1.005, cost of draft for \$1.

\$500 \div 1.005 = \$497.51 +, face of draft. Ans.

EXPLANATION.— Dividing the cash invested in the draft, \$500, by the cost of a draft for \$1, or \$1.005, gives the number of dollars in the draft, or the face of the draft, \$497.51 +.

EXAMPLES FOR DRILL.

1. With exchange $\frac{3}{4}\%$ discount, find the face of a draft that can be bought for \$587.60.

2. A commission merchant sells a consignment for \$15000, reserves his commission of $1\frac{1}{2}\%$ and charges amounting to \$275.45; he then remits proceeds by 60-day draft purchased at $\frac{1}{4}\%$ premium, discount at 1% per month, without grace. Find the face of the draft.

3. Smith deposits \$4750 with a bank, taking in exchange for his money a 20-day draft on New Orleans, bought at $\frac{1}{2}\%$ premium, and discounted at 12% without grace. Find the face of the draft.

4. Mr. A invested \$6000 in Chicago exchange at 60 days, brokerage $\frac{1}{4}\%$, course of exchange $\frac{1}{2}\%$ below par, and time discounts 1% per

month. Thirty days after he discounted his draft at a bank at $C\%$. Required the proceeds.

5. When discounts were 1% per month without grace, a 60-day draft for \$600 was obtained for \$586.50. Required the course of exchange.

SUGGESTION.—Find the cost of a draft for \$1 and add to this the bank discount for 60 days, or 2% . The result is the course of exchange.

RATIO AND PROPORTION.

454. We have already seen (Art. 127 — 131) that the relation, or ratio between two quantities of the same kind is found by dividing one by the other.

455. **Ratio** is the result of dividing one quantity by another of the same kind.

Ratio is indicated by the sign “:”; thus the expression “12:4” is read, “12 is to 4,” and implies the ratio of 12 to 4.

456. Two quantities joined by the ratio sign, “:” are called a **Couplet**.

457. **The Antecedent** is the first term of the couplet and is the dividend.

458. **The Consequent** is the second term of the couplet and is the divisor.

459. The *Ratio* is the quotient arising from dividing the Antecedent by the Consequent.

ILLUSTRATION.—In the expression $\$24 : \$8 = 3$; \$24 is the Antecedent, \$8 is the Consequent, and 3 is the Ratio.

460. A **Simple Ratio** is the ratio of two numbers; as 12 : 6.

461. A **Compound Ratio** is the product of two or more simple ratios.

A compound ratio may be reduced to a simple ratio, by obtaining the product of the antecedents, for the antecedent of the simple

ratio, and the product of the consequents, for the consequent of the simple ratio.

As, $\left\{ \begin{array}{l} 8 : 4 \\ 12 : 6 \end{array} \right\} = 8 \times 12 : 4 \times 6$ is a compound ratio and reduced to a simple ratio is $96 : 24$.

462. Ratio is based upon the following

PRINCIPLES.

- I. *Ratios subsist between like numbers only.*
- II. *The Antecedent is equal to the Product of the Consequent and the Ratio.*

NOTE.—The general principles of Division (Art. 61), also apply to the terms of Ratio.

463. A **Simple Proportion** is the expression of equality between two simple ratio couplets, and contains four terms. The ratio couplets are usually connected by the double colon ($::$); thus, $12 : 4 :: 21 : 7$; the sign " $::$ " implies equality, and the proportion is read, "12 is to 4 as 21 is to 7." Sometimes the sign of equality [=] is used instead of the double colon, and the ratio divisions are expressed in the form of a fraction. Thus, the proportion $15 : 5 :: 18 : 6$, may be written, $\frac{15}{5} = \frac{18}{6}$.

464. The terms of a proportion are designated by number, as First, Second, Third, and Fourth. The First and Fourth terms of a proportion are called its **Extremes**, and the Second and Third terms its **Means**. Thus, in the proportion " $18 : 6 :: 15 : 5$," 18 and 5 are the Extremes, while 6 and 15 are the Means.

465. From Prin. 2, Art. 462, it is evident that the First Term of every proportion is equal to the Second Term \times Ratio, and that the Third Term of every proportion is equal to the Fourth Term \times Ratio. The product of the Extremes of any proportion is therefore equal to the Second Term \times Ratio \times Fourth Term of that proportion, and the product of the Means of any proportion is equal to the Second Term \times Ratio \times Fourth Term of that proportion. Since the Means and the Extremes include the same factors, their respective products are equal. Hence the

GENERAL LAW.

In every proportion, the product of the Means is equal to the product of the Extremes.

466. Three Terms of a Proportion given, to find the Remaining Term.

ILLUSTRATIVE EXAMPLE.

$$28 : 7 :: 36 : (?)$$

SOLUTION BY ANALYSIS.

$28 \div 7 = 4$, the Ratio. Since 36, the Third Term = Ratio \times Fourth, or required term, $36 \div 4 = 9$, the required term. Hence, $28 : 7 :: 36 : 9$.

SOLUTION BY CANCELATION.

$$\begin{array}{l|l} 28 & 7 \\ (?) & 36 \end{array} \quad \begin{array}{l} 9 \text{ req. term.} \\ \text{Ans.} \end{array}$$

EXPLANATION.—Write the means (or given mean) on one side of a vertical line, and the extremes (or given extreme) on the other.

Divide by the single term. The result will be the required term, since the product of the extremes equals the product of the means. It is best to adhere to some permanent arrangement of the terms, as the following:—

(Extremes.)	(Means.)
First Term.	Second Term.
Fourth Term.	Third Term.

Solve the following examples by both analysis and cancelation.

EXAMPLES FOR DRILL.

- | | |
|-----------------------------|-----------------------------|
| 1. $54 : (?) :: 120 : 20$. | 3. $240 : 48 :: 165 : (?)$ |
| 2. $68 : 4 :: (?) : 3$. | 4. $(?) : 142 :: 98 : 14$. |

STATING PROPORTIONS.

In many kinds of practical problems, the quantities sustain to each other the relations of proportion.

These proportions may all be stated by means of the following

RULE.

Write for the First and Second Terms of the proportion, the two like quantities given in the problem. Then write the remaining quantity for either the Third or the Fourth Term, according as it is larger or smaller than the required term.

NOTE.—If the First Term be larger than the Second, it follows that the Third must be larger than the Fourth.

ILLUSTRATIVE EXAMPLE.

If 72 hats cost \$90, what cost 60 hats ?

SOLUTION.

Hats. Hats.

72 : 60 :: \$90 : (?)

\$ ~~72~~ | \$0 5

(?) | \$0 15

15 × 5 = \$75. Ans.

EXPLANATION.—Take the two quantities of hats for the first couplet, 72 : 60; then as the given value of 72 hats, \$90, is greater than the required value of the 60 hats, and as the third term of the proportion must be greater than the fourth, we write \$90 for the third term, and find the fourth term as in Art. 453.

NOTE.—In practice, the formal statement of the proportion is unnecessary. Simply arrange the terms on either side of the vertical line, and cancel.

EXAMPLES FOR DRILL.

1. If 125 bu. of wheat cost \$50, what would 38 bu. of wheat cost ?
2. If 14 lb. of sugar cost 63¢, what will 22 lb. cost ?
3. If 15 horses cost \$1200, what will 37 horses cost ?
4. If \$10.50 is the interest on \$270, what would be the interest on \$450 for the same time and at the same rate ?
5. A stockholder receives a dividend of \$600 on \$7500 of stock. What would he have received, if he had owned \$11550 of the same stock ?
6. If a post 15 ft. long casts a shadow of 6 ft., what length of shadow will a post 18 ft. long cast at the same time ?
7. A bankrupt owed \$12000, and his assets were \$7000. How much would a creditor receive on a claim of \$3000 ?
8. A grocer paid \$8 for berries at the rate of 18 qt. for \$1.25. How many quarts did he buy ?
9. On property assessed at \$14500, the tax is \$174. What would be the tax on property assessed at \$17850 ?
10. If 14 men cut 420 cd. of wood, how many men will it take to cut 1500 cd. in the same time ?
11. If 7 lb. 8 oz. of butter cost \$1.60, how much will 4 lb. 14 oz. cost ?
12. If a person completes a journey in 36 da. of 8 hr. each, how many days of 9 hr. each would it require ?

13. If the rate of freight is 12¢ a bushel for 840 miles, what will be the proportional rate for 784 miles?

14. If it requires 45 men 28 da. to complete a certain contract, in what time would 63 men complete it?

467. To separate a number into parts proportional to two or more given numbers.

ILLUSTRATIVE EXAMPLE.

Divide \$1050 into sums proportional to 3, 5, and 7.

SOLUTION.

$$3 + 5 + 7 = 15.$$

$$\frac{3}{15} (= \frac{1}{5}) \text{ of } \$1050 = \$210, \text{ 1st sum.}$$

$$\frac{5}{15} (= \frac{1}{3}) \text{ of } \$1050 = \$350, \text{ 2d sum.}$$

$$\frac{7}{15} \text{ of } \$1050 = \$490, \text{ 3d sum.}$$

EXPLANATION.—The sum of the proportional numbers is 15. Then the smallest sum required will be $\frac{3}{15}$, or $\frac{1}{5}$, of \$1050. The next sum required will be $\frac{5}{15}$, or $\frac{1}{3}$, of \$1050; and the third sum will be $\frac{7}{15}$ of \$1050, or \$490.

RULE.—*Take such fractional parts of the quantity to be divided as the given proportional numbers are, respectively, of their sum.*

EXAMPLES FOR DRILL.

1. The gross freight charges on a carload of goods are \$89.70. The car was hauled over two roads, one 240 mi. long and the other 360 mi. long. What sum should each road receive?

2. John's money is $\frac{5}{6}$ of James's money, and both have \$29.70. What sum has each?

SUGGESTION.—The money is divided in the proportion of $\frac{5}{6}$ to 1.

3. Three packages weigh respectively 720 lb., 540 lb., and 900 lb. The total freight charges were \$28.80. Find the charges on each package.

4. Three men, A, B, and C, performed a contract for which they received \$126.90. A worked $5\frac{1}{4}$ da., B $6\frac{1}{2}$ da., and C $6\frac{1}{4}$ da. What sum should each receive?

5. \$1250 was paid for 4 cables, the first 240 ft., the second 180 ft., the third 160 ft., and the fourth 380 ft. What was the cost of each cable?

COMPOUND PROPORTION.

468. A Compound Proportion is one containing compound ratios.

Thus, $\left\{ \begin{array}{l} 4 : 6 \\ 2 : 3 \\ 5 : 2 \end{array} \right\} :: 10 : 9$, is a compound proportion. Either one or both of the ratio-couplets may be compound.

469. In stating compound proportions do not reduce the compound ratios to simple ones but place the compound terms in the same positions they would occupy were they simple terms.

ILLUSTRATIVE EXAMPLE.

If 24 men in 8 da. of 10 hr. each, build 1600 rd. of fence, how many rods can be built by 36 men in 15 da. of 8 hr. each?

SOLUTION.

Extreme .	Means.
24	3 \$6
\$	15
10	\$
(?)	1600
	80

EXPLANATION.—The number of rods built under the respective conditions is proportional to the men, the days, and the hours in each condition, or (Art. 461) to the product of the terms of these ratios.

$$15 \times 3 \times 80 = 3600 \text{ rods. Ans.}$$

EXAMPLES FOR DRILL.

1. If 8 men in 10 da. dig a ditch 120 rd. long, how many rods of the same ditch should 12 men dig in 15 da.?

2. If \$3000 loaned for 2 yr. 6 mo. produce \$600 interest, how much should \$3500 produce in 1 yr. 3 mo.?

3. If the interest on \$1200 for 3 mo. at 6% is \$18, at what rate per cent per annum will \$800 produce \$12 in 60 da.?

4. A mason receives \$300 for building a stone wall 80 ft. long, 10 ft. high and $1\frac{1}{2}$ ft. thick. What should he receive for building a stone wall 60 ft. long, 18 ft. high, and 2 ft. thick?

5. If \$1400 is earned by 15 men in 30 da., working 10 hr. a day, how much can 18 men earn in 27 da. by working 9 hr. a day?

6. If a bin 6 ft. long, 4 ft. wide, and 4 ft. deep will hold 100 bu. of wheat, how deep must a bin be that is 9 ft. long, and 6 ft. wide so that it will hold 270 bu.?

7. The freight on 9240 lb. for 900 mi. was \$46.20; what would be the freight on 25200 lb. of the same kind of goods for 300 mi., if a proportional rate was charged?

PARTNERSHIP.

470. A Partnership is an association of two or more persons to place their money, skill, or labor, in some enterprise or business, and divide the profits, and share the losses according to agreement.

Such an association is generally called a **Firm**, or **House**, and the persons who form the partnership are called **Partners**.

471. Articles of Copartnership, or a Partnership Contract, is a written instrument, setting forth the agreement between the partners, stating the amount of each partner's investment, the time during which the partnership is to continue, the allowance to be withdrawn by each partner, the manner in which the profit or loss is to be divided, and such other particulars as may be considered expedient.

472. The Capital, or Investment, is the money or other property invested in the business.

473. The Assets, or Resources, of a firm are its entire property and all debts due the firm.

474. The Liabilities of a firm include all the debts and other obligations the firm owes to its creditors.

The Net Capital, or Present Worth, of a firm is the difference between the total resources and the total liabilities.

475. The Insolvency of a firm is the excess of its liabilities over its resources.

476. The Net Gain is the excess of the total gains over the total losses during a given period.

477. The Net Loss is the excess of the total losses over the total gains for a given period.

478. To distribute gains and losses and to determine partnership balances.

EXAMPLES FOR DRILL.

1. A firm's capital, on commencing business Jan. 1, 1891, was \$15655, the net gain for the year was \$4580. What was the firm worth at the end of the year?

2. A firm's capital, on commencing business June 1, 1893, was \$13425, the net loss for the year was \$1584. What was the net capital of the firm June 1, 1894?

3. A firm invested \$15000, Feb. 1, 1892, and on Feb. 1, 1893, their present worth was \$17384. Find the net gain.

4. Jan. 26, 1893, a firm invested \$13325, and on Jan. 26, 1894, the firm's present worth was \$12381. Find the net loss.

5. The resources of a firm are \$16482, the liabilities, \$4584. What is the present worth?

6. A firm invested \$8424.50, Jan. 15, 1894; the insolvency of the firm on Jan. 15, 1895, was \$4325.75. What was the net loss of the firm?

7. Smith & Johnson commenced business with the following resources: Cash \$9800, merchandise \$14500; they owed on personal accounts \$3600, and on notes \$6500. What was the net capital of the firm?

8. A firm invested on commencing business \$6500 cash, real estate valued at \$2500, notes amounting to \$2300. At the end of a year they had on hand, cash \$7500, real estate valued at \$4500, notes amounting to \$2450, and personal accounts due them \$2350; and owed on personal accounts \$2650, and on notes \$1300. What was the net capital of the firm at the end of the year, and amount of gain or loss?

9. Jas. S. Simons and J. B. Snow entered into partnership, the former investing \$3600, and the latter \$3000. At the end of a year they had merchandise on hand worth \$4000, cash on hand \$3500; bills receivable \$2400, and personal accounts due the firm \$1678; they owed J. B. Crosby \$980, and R. H. Hopkins \$700. What was the net gain of the firm?

479. To apportion the gain or loss when each partner's investment has been employed for the same time.

ILLUSTRATIVE EXAMPLE.

A and B engage in business; A invests \$6000, and B \$5000. The gain for the year was \$2200, which was divided between the partners in proportion to their investments. What was each partner's share of the gain?

SOLUTION.

$\$6000 + \$5000 = \$11000$, total investment. If \$11000 investment produces \$2200 gain, \$1 will produce $\$2200 \div \11000 , or \$.20 gain, and \$6000 investment will produce \$1200 gain, and \$5000 will produce \$1000 gain. Or, find each partner's share of the gain or loss, by Proportion, Art. 454.

EXAMPLES FOR DRILL.

1. W. F. Kingman and F. Brown are partners. The former invested \$7500 and the latter \$5500. The gain for the year was \$3900. Find each partner's share of the gain if divided in proportion to their respective investments.

2. A, B, and C formed a partnership. A investing \$5000, B \$8000, and C \$12000. The net gain for the year was \$7500, which was divided in proportion to the partners' investments. What was each partner's share of the gain, and what was the present worth of each partner?

3. Two men engaged in business with a capital of \$25000, one partner furnishing \$14000, and the other the remainder. At the end of the year they are worth \$37500. What would be each partner's share of the gain?

4. On Jan. 1, 1894, the firm of Johnson & Hunt had the following resources: Cash \$15400, bills receivable \$10400, and personal accounts \$7100. The liabilities of the firm were: Bills payable outstanding \$2500, personal accounts outstanding \$4500. Johnson's investment was \$11500, and Hunt's investment \$10500. What was the gain or loss, and the share of each?

5. A and B are partners for one year. A invests \$6400, B \$5600, and they agree to share the gain or loss in proportion to investment. At the end of the year they had on hand, cash \$4500, merchandise \$3600,

personal accounts, due the firm \$9900, and their total liabilities \$3000. Find the gain of each.

6. L. A. Palmer and J. S. Pierce engage in the grain business. Palmer invests 10000 bu. of corn invoiced at 54¢ per bushel, and Pierce invests, cash \$1200, personal accounts due him amounting to \$750, real estate worth \$2000, and a warehouse valued at \$2050. If a gain of \$3600 is divided in proportion to their investments, find how much each should receive.

480. To divide the gain or loss in proportion to investments when invested at different times.

ILLUSTRATIVE EXAMPLE.

A and B engaged in business Jan. 1, 1894, each investing \$4000. On April 1, A made an additional investment of \$1000, and B withdrew \$500; Oct. 1, A withdrew \$500 and B invested \$1500. How should a gain of \$1725 be divided?

SOLUTION.

From Jan. to Apr. 1 = 3 mo.	Then \$4000 × 3 = \$12000 for 1 mo.
April 1 investment,	1000
From Apr. 1 to Oct. 1 = 6 mo.	Then \$5000 × 6 = \$30000 for 1 mo.
October 1 withdrawal,	500
From Oct. 1 to Jan. 1 = 3 mo.	Then \$4500 × 3 = \$13500 for 1 mo.
A's investment =	\$55500 for 1 mo.
\$55500 ÷ 12 = \$4625, A's average investment throughout the year.	
From Jan. 1 to Apr. 1 = 3 mo.	Then \$4000 × 3 = \$12000 for 1 mo.
April 1 withdrawal,	500
From Apr. 1 to Oct. 1 = 6 mo.	Then \$3500 × 6 = \$21000 for 1 mo.
October 1 investment,	1500
From Oct. 1 to Jan. 1 = 3 mo.	Then \$5000 × 3 = \$15000 for 1 mo.
B's investment =	\$48000 for 1 mo.
\$48000 ÷ 12 = \$4000, B's average investment throughout the year.	
\$4625 + \$4000 = \$8625, Firm's average investment.	
\$1725 ÷ 8625 = .20, or 20%.	
\$4625 × .20 = \$925, A's gain.	\$4000 × .20 = \$800, B's gain.

EXPLANATION.—A's investment Jan. 1 (\$4000) remained unchanged until Apr. 1, or for 3 mo.; and \$4000 invested for 3 mo. = an investment of \$12000 for 1 mo.

On Apr. 1 A invested \$1000, making his investment \$5000, which remained unchanged to Oct. 1, or for 6 mo., and \$5000 invested for 6 mo. = an investment of \$30000 for 1 mo.

On Oct. 1 A withdrew \$500, leaving \$4500 in the business, which remained unchanged to the end of the year, or for 3 mo., and an investment of \$4500 for 3 mo. = an investment of \$13500 for 1 mo.

Then A's total investment = $\$12000 + \$30000 + \$13500 = \55500 for 1 mo., or an average investment of $\$55500 \div 12 = \4625 for the year.

Obtain B's average investment in the same manner, which was \$4000. Then divide the gain for the year in proportion to their average investments, either by the method given, or by Proportion, Art. 454.

EXAMPLES FOR DRILL.

1. A and B engaged in business for 1 year, and gained \$2550. On Mar. 1, 1893, A invested \$3000, and B \$2000. Sept. 1, A withdrew \$600, and B invested \$800. If the gain was divided in proportion to their average investments, what was each partner's share of the gain?

2. Martin and Wilson were partners in business, each investing \$5000, on Jan. 1, 1893. Apr. 1, 1893, Martin invested \$1000, and Wilson withdrew \$500. Oct. 1, Martin withdrew \$800, and Wilson invested \$900. On Jan. 1, 1894, the net gain was \$5000. What was each partner's share?

3. A and B engaged in business on May 1, 1892, A investing \$6000, and B \$4000. On July 1, A invested \$2500 additional; on Jan. 1, 1893, he withdrew \$2000. On Sept. 1, B invested \$3000 additional; on Feb. 1, he withdrew \$1500, and on April 1, he put in \$1000. On Oct. 1, C was admitted as a partner, investing \$7000. At the end of one year the net gain was found to be \$6400. If the gain was divided in proportion to their average investments, what was each partner's share?

4. On Jan. 1, 1891, S. H. Anson and R. S. Anderson formed a copartnership for one year, each investing \$10000. On May 1, Anson invested \$1000 additional, and Anderson withdrew \$500. On July 1, each invested \$800, and on Oct. 1, Anson withdrew \$1000, and Anderson invested \$650. The firm's resources at the end of the year were \$25500, and their liabilities \$1500. What has been the net gain or loss, and the share of each partner if divided according to average investment?

5. A and B form a copartnership for two years, A investing \$14000, and the firm assuming his liabilities of \$3000. B invested \$10000. At the end of the first year, A invested \$2000 additional, and B, \$4000. At the end of the next six months, A withdrew \$1500, and B, \$1800. At the end of two years, the net loss was found to be \$5600. If the loss was shared in proportion to the average investment, what was the loss of each partner?

BANKRUPTCY.

481. A **Bankrupt**, or **Insolvent**, is a person who fails in business and has not sufficient assets to meet his liabilities.

482. The **Assets** of a bankrupt are his entire property, including debts due him. *Nominal Assets* are those which are not convertible into cash immediately, unless sold for less than their stated value.

483. The **Liabilities** of a bankrupt are the obligations due by him to his creditors.

484. An **Assignment** is the transfer of the property of a bankrupt to a certain person who has full control of it for the benefit of the creditors. This person is called the **Assignee**. Should a corporation become bankrupt, a **Receiver** is appointed to take charge of the business of the bankrupt corporation.

485. A **Statement**, or **Schedule**, is a list setting forth the liabilities and assets of the bankrupt, giving the name of each creditor or debtor of the firm, their place of business or residence, and the amount due to or from each.

486. A **Dividend** is the amount or portion of the assets which is paid by the assignee to the creditors.

ILLUSTRATIVE EXAMPLE.

A lumber-dealer failed in business with the following assets and liabilities. Assets: Cash, \$4500.50, real estate, \$5648.50, lumber on hand, \$19660, personal accounts due him, \$2370. Liabilities: Notes outstanding, \$2500, owes J. W. McDowell, \$11400; J. S. Brown, \$14560; Chas. Moore, \$7050, and the expenses of the assignment were \$220. How much would each creditor receive?

SOLUTION.

$\$4500.50 + \$5648.50 + \$19660 + \$2370 = \$32179$, total assets.

$\$32179 - \$220 = \$31959$, net assets.

$\$2500 + \$11400 + \$14560 + \$7050 = \$35510$, total liabilities;
then $\$31959 \div \$35510 = \$0.90$ the amount to be paid on every dollar
of debt.

$\$2500 \times \$0.90 = \$2250$, dividend on notes.

$\$11400 \times \$0.90 = \$10260$, dividend to J. W. McDowell.

$\$14560 \times \$0.90 = \$13104$, dividend to J. S. Brown.

$\$7050 \times \$0.90 = \$6345$, dividend to Chas. Moore.

EXAMPLES FOR DRILL.

1. Warr, Dell & Co. failed in business for \$120000, their assets amounting to \$95670. How much would each creditor receive on the dollar, if the expenses of settlement were \$5670?

2. A bankrupt's estate is worth \$25000, his debts amount to \$32000. How much could be paid on the dollar, and what would a creditor get whose claim is \$3200, if the assignee's charge is 2% of the assets?

3. Hewett & Co. placed in the hands of an assignee for the benefit of their creditors, a stock of goods which sold for \$25000, and personal property which sold for \$8500. Their entire liabilities were \$44500. If the assignee's charges were 5% of the assets, what per cent of the indebtedness was paid?

4. Carter, French & Co. failed, owing A \$13340, B \$7680, and C \$10420. Their assets were \$14150, exclusive of the real estate, and expenses of settling were \$1500; they owed their employees \$2500, which was to be paid in full. If the real estate brought \$12000, what per cent should be paid, and how much would the other creditors receive?

5. W. C. Phelps's claim against an insolvent firm was \$3600, and J. C. Owen's claim was \$4000. The assets remaining after deducting the expenses of the assignment, were \$7500. If the total liabilities of the firm were \$10000, how much would Phelps and Owen receive?

6. The Empire Starch Co. failed for \$361000, with assets that were available of \$125648. If the assignee's fee was \$3480, what should J. R. Howard receive for a claim of \$36100?

7. The stock of a bankrupt merchant was sold by the assignee for \$10500; the assignee also collected on notes, \$3187, and on personal accounts, \$5000; the total liabilities of the firm were \$28650, and the assignee's charge was \$924. What dividend can be paid, and what should F. A. Hopper receive whose claim is \$3480?

GENERAL REVIEW PROBLEMS.

1. A cashier's receipts and disbursements for the week were as follows: cash on hand \$125; receipts for Monday \$250, disbursements \$175; Tuesday, receipts \$165, disbursements \$210; Wednesday, receipts \$340, disbursements \$95; Thursday, receipts \$164, disbursements \$92; Friday, receipts \$350, disbursements \$425; Saturday, receipts \$956, disbursements \$75. Find the balance of cash on hand at the end of the week.

2. A lady finds that if she buys a certain number of yards of lace at 65¢, she will have \$3.75 left, and that if she pays 82¢ per yard she will lack \$2.71 of having enough money to pay for it. How many yards did she wish to buy and how much money did she have?

3. A firm has cash on hand \$7645.65; Mdse. as per invoice \$16-741.25; solvent credits amounting to \$8567.95; and store fixtures \$658.90. The firm owes on notes \$1946.30, and on accounts \$9,-438.75. What is the net capital of the firm?

4. A farmer harvested three fields of grain containing respectively, 50, 75, and 90 acres. The first field yielded 645 bu., the second 816 bu., and the third 1420 bu. What was the average yield per acre?

5. A lot 198 ft. long 144 ft. wide is to be fenced. What is the length of the longest boards that can be used for the panels, in order that there may be a whole number of panels on both the sides and the ends of the lot?

6. At the beginning of the year a firm's resources were as follows: Cash \$1956; Mdse. \$9658; Notes and accounts \$3587. The firm owed on notes \$2467, and on accounts \$3586. At the end of the year the firm's resources were Cash \$2172, Mdse. \$5876, Notes and accounts \$8349. At this time the firm owed on notes \$1928, and on accounts \$2768. Did the firm lose or gain during the year, and how much?

7. The records of the U. S. Weather Bureau show that the mean temperature for the month of January at a certain station was as follows:

For 5 days in the month, 26.5 degrees.

" 7 " " " " 28.3 "

" 12 " " " " 31.2 "

And for the remainder of the month 34.6 degrees. Find the average mean temperature for the month.

8. A grocer mixes 24 lb. of tea at 75¢ with 42 lb. at 93¢. At what price per pound must he sell the mixture to gain \$12.50?

9. What is the least number that can be divided by 18, 24, 30, and 42, and leave a remainder of 13 after each division?

10. When hogs were worth $6\frac{1}{4}$ ¢ per pound, a farmer exchanged 6 hogs, average weight 275 lb., for 150 bales of hay, average weight 125 lb. What did the hay cost him per ton?

11. On May 1st, Smith owed his grocer on account \$64.75. On May 15th, he bought a bill of goods on account amounting to \$16.25, and on the 20th another of \$28.65. On the 23d he sold the grocer on account butter and eggs to the amount of \$7.95, and on the 27th paid \$17.75. What does he owe the grocer on June 1st?

12. Bought a farm for $\frac{1}{4}$ less than its value and sold it for $\frac{1}{3}$ more than its value and gained \$3500. What did I pay for the farm?

13. A lady finds that a dress including the making will cost her \$30.35 if she selects material worth \$1.10 per yard; but if she selects material worth 95¢ per yard the cost of her dress will be \$27.95. How many yards of goods in the dress, and what is the cost of making?

14. There are three walls whose lengths are respectively 1769 ft., 2059 ft., and 2639 ft. Required the length of the longest pole that will exactly measure each of them.

15. A board is $17\frac{3}{4}$ in. wide at one end and $11\frac{3}{8}$ in. wide at the other end. What is the average width of the board?

16. If the above-described board is 26 ft. long, how many square feet does it contain?

17. If the board is $3\frac{1}{2}$ in. thick, what is it worth at \$2.50 per 100 bd. ft.?

18. It requires 402 rd. of fencing for a lot 125 rd. long. How many acres in the lot?

19. \$128.25 was paid for a pile of wood 72 ft. long, 8 ft. wide and 6 ft. high. What was the wood worth per cord?

20. At the beginning of the day there was in a merchant's cash-drawer \$56.95. There were recorded cash sales during the day amounting to \$246.75. There was also an unknown amount of cash sales made that were unrecorded. During the day \$28.65 was taken from the drawer, and at the closing hour, the cash in the drawer amounted to \$286.50. What was the amount of unrecorded sales?

21. A man spent $\frac{1}{4}$ of his money for a coat. Had the coat cost \$5 more it would have required $\frac{1}{3}$ of his money to pay for it. What did the coat cost?

22.

CHICAGO, Mar. 6, 1895.

ARKWRIGHT, MASON & Co.,

Sold to BANISTER & SONS.

Terms 60 da. 2%. 30 da. 5%, cash 10%,

5 doz. Wright Apple Parers.....	@ \$16.50	20%.
3½ doz. No. A. Diston Saws.....	@ \$15	Net.
4 doz. Lawn Mowers.....	@ \$37.50	15%.
7 doz. Garden Rakes.....	@ \$ 4.50	15%.
15 Grindstones with mountings, ea.....	\$3.75	Net.

Find amount of above invoice if bought (a) for cash, (b) at 20 da., (c) at 45 da., (d) at 90 da.

23. At the beginning of the year a merchant has resources to the amount of \$26795, and owes debts to the amount of \$9467. At the end of the year he has gained \$7579, and has resources to the amount of \$29578. What sum does he owe at the end of the year?

24. A lot of land is 360×484 ft.; what is it worth at \$75 per acre?

25. The water that falls upon a roof 45×66 ft. is conveyed by pipes to a cistern 10 ft. 6 in. in diameter. After a 7-inch rain, what will be the depth of the water in the cistern?

SUGGESTION.—The depth of the water will be as much greater than 7 inches as the area of the roof is greater than the area of the bottom of the cistern.

26. At \$17.50 per M. what cost the planking for a barn floor 65×56 ft., the planking being 3 in. thick?

27. From a piece of land 246×472 rd., a lot 120×180 rd. was sold. What decimal part of the whole (correct to four places) was sold and what was the value at \$125 per acre, of the part remaining?

28. Allbright & Co. bought 26580 lb. of wool at $15\frac{3}{8}\%$ and sold it at $16\frac{3}{4}\%$. What was their whole gain and their per cent of gain (correct to hundredths of a per cent)?

29. On Mar. 2, I bought a horse for \$125. On Aug. 23, I sold him for \$140. My gain was equivalent to what rate of interest on my investment?

30. Smith receives \$8 per month ground-rent for a piece of real estate. If money is worth 6%, what is the "capitalization," or value of the property?

31. If the average depth of a stream is 26 in., average width 15 ft., and its current velocity 2 ft. per second, how many barrels of water per hour would the stream yield?

32. Mr. Wilson owned a piece of city property which he rented for \$28 per month. His insurance and water rates amounted to \$60 per year, and his annual repairs to \$25. He sold the property for \$3000 and invested the money in U. S. 5 per cent bonds at $86\frac{1}{2}$, brokerage $\frac{1}{8}$, the lowest denomination of bonds being \$100. Was his annual income increased or diminished, and how much?

33. What is the difference between the true and bank discount on \$640 for 90 da. (without grace) at 6%?

34. The present worth of a certain debt at 6% and due in 7 mo. 20 da. is \$257.40. What is the amount of the debt?

35. A circular tank is $10\frac{1}{2}$ ft. in diameter and 16 ft. deep. How many tons of water will it hold?

SUGGESTION.—The weight of the water is equal to the total pressure upon the bottom.

36. Arthur Brown was born May 29, 1882. What sum should be placed at compound interest at 6% for Arthur's benefit, on July 20, 1895, in order that he may have \$5000 due him when he becomes of age, interest compounded semi-annually?

37. Sent my agent \$1249 to be invested in butter at 20¢ after deducting his commission at 2% and other charges at \$25. How many pounds of butter were bought?

38. When money was worth 8%, Smith loaned Davis \$850 for 75 da. When money is worth 6%, for what time would Davis have to loan Smith \$500 to cancel the obligation?

39. Bought an invoice of boots at a net cash cost of \$2.40 per pair. At what price should I mark them, in order that I may give a discount of 20% from the marked price, and still net $16\frac{2}{3}\%$ on the cost?

40. If it cost \$150 to fence a square field at 75¢ per rod, what will it cost to plow the field at \$1.25 per acre?

41. By buying Illinois Central at $115\frac{1}{4}$, brokerage $\frac{1}{8}$, and selling at $118\frac{1}{2}$, brokerage $\frac{1}{4}$, I gained \$201.25. What sum did I pay for the stock?

42. When it is 11:55 p. m. at Baltimore, what is the time (standard time) at San Francisco?

43. An express train running at the rate of 45 mi. per hour, runs how many feet per second?

44. What is the difference between the ordinary and the accurate interest on \$5000 for 20 da. at 10%?

45. By buying a piece of property at 8% less than its value and selling it at 10% more than its value, I gained \$720.36. What sum did I pay for it?

46. A bicycle wheel 56 in. in diameter makes 60 revolutions per minute. At a rate of how many miles per hour is the bicycle moving?

47. Add the following fractions, and reduce the result to a decimal; correct to five places: $\frac{2}{3}, \frac{5}{9}, \frac{7}{12}, \frac{5}{24}, \frac{5}{18}$.

48. Reduce each fraction in the preceding example to a decimal; correct to five places and add the result.

49. Baker & Sons received \$690.34 as net proceeds on a consignment of butter sold at 20¢ per pound. If the commission was 2% and the other charges \$8.40, how many pounds of butter were sold?

50. A railroad passes through Mr. Miller's farm for a distance of 130 rd., the right of way being 75 ft. wide. What is the land taken by the railroad worth at \$60 an acre?

51. Jones bought a town lot for \$200 on the following terms: he is to pay \$20 in cash and the balance in instalments of \$20 per month; also 6% interest on the unpaid portion of the purchase-price until the prop-

erty is paid for. If the instalments were paid according to agreement, what was the total cost of the property?

52. What is the difference between the simple and compound interest on \$750 for 2 yr. 9 mo. 15 da. at 8%; interest compounded semi-annually?

53. If I sell bicycles at \$90 each on a commission of 15%, *remitting proceeds in advance*, what per cent do I make on my investment?

54. The cost of insuring my barn at $\frac{3}{4}$ of its value for 3 yr. at an annual rate of $1\frac{1}{4}\%$, amounts to \$21.60; what is the value of my barn?

55. The height of a cask is 6 ft. 3 in. and its average diameter 5 ft. 10 in.; how many bushels of wheat will it hold?

56. Bought through a factor 30000 bu. of wheat at 72¢, commission $1\frac{1}{2}\%$. Afterward the agent sold the wheat at 75¢, commission 2%, paid charges \$23.40 and, after deducting both commissions and the charges, remitted balance by draft purchased at $\frac{3}{8}\%$ discount. Required, my net gain on the speculation.

57. The entire cost of publishing an edition of 20000 copies of a book amounted to \$23447. If the book is retailed at \$2 through agents who receive a commission of 30%, and the author receives a royalty of 10% on the wholesale price, what sum will the publishers gain on the entire edition?

58. The true discount on a certain debt for 3 mo. 12 da. at 10%, is \$6.80. What is the face of the debt?

59. If I buy goods at a discount of 15% and 10% off list-price, and sell them at list-price, what per cent do I gain?

60. A note of \$720 dated Jan. 6, 1893, bearing interest at 10% and maturing June 18, 1895, is discounted at a bank Mar. 2, 1895, at 12%; required the proceeds of the note.

61. A owns a house and lot which he rents at \$12 a month, and expends 10% of the rent yearly for repairs and other expenses. He sells the property for \$1500 and invests the money in 8% bonds at $74\frac{3}{4}$, brokerage $\frac{1}{4}$. Is his annual income increased or diminished, and how much?

62. A piece of land is 75 rd. wide and contains 45 acres. What will it cost to fence it at \$1.35 per rod?

63. The bank discount on a note for 63 days at 8% is \$12.60. Find the face of the note.

64. Find the weight in Avoir. pounds of \$1000000 in gold.

65. A steam boiler is fed by a pipe connecting with a reservoir 150 ft. above the boiler. What pressure of steam will stop the flow of the water into the boiler?

66. On March 22, 1893, Albert Shaw settled his account with A. Bailey & Co., giving his note with interest at 10% for the balance, amounting to \$275.60. July 7, 1893, Shaw paid \$75, and on Dec. 20, 1893, \$50. If "rests," or settlements, are had every six months, what does Shaw owe on the note Sept. 26, 1894?

67. A broker bought for a customer a certain number of shares of Spring Valley at $93\frac{1}{4}$, brokerage $\frac{1}{8}$, and sold them at $102\frac{1}{8}$, brokerage $\frac{1}{4}$. The whole brokerage amounted to \$71.25. What did the customer gain by the speculation?

68. At \$17.20 per M. what cost the lumber for 500 boxes, each 3 ft. long, 2 ft. 6 in. wide, and 1 ft. 9 in. high, lumber 1 in. thick, no allowance for corners?

69. A farmer sells a merchant 7 wagon-loads of potatoes, weighing respectively, 3567 lb., 3690 lb., 3375 lb., 3742 lb., 3827 lb., 3660 lb., and 3146 lb. If the wagon weighs 1568 lb. and the potatoes are worth 65¢ a bushel, how much money should the farmer receive?

70. When exchange is $\frac{1}{2}\%$ premium and time discounts are 6%, required the proceeds of a 90-day draft for \$570, brokerage $\frac{1}{4}\%$.

71. How many silver dollars can be made from a silver brick 95% fine and weighing 50 lb. Avoir.?

SUGGESTION.—Divide the pure silver in the brick by the pure silver in a dollar.

72. Smith borrowed \$5000 for 5 yr. at 6%, agreeing to pay the interest semi-annually and also to pay 10% interest on all overdue interest instalments. The first three interest-payments were made when due, and the remainder were unpaid. What was the whole sum due when the note matured?

73. If I pay \$1.20 per cord for sawing eight-foot wood into two-foot lengths, what should I pay per cord for sawing four-foot wood into the same lengths?

74. A holds a negotiable note for \$400, dated June 4, 1893, bearing interest at 8% and due Jan. 1, 1896. What sum should B give for the note on May 10, 1895, in order that he may gain 10% interest on his investment?

75. Sold two lots for \$1800 each; on one I gained 25% of its cost, and on the other I lost 25% of its cost. Did I lose or gain on both transactions, and how much?

76. How many bricks, each $8 \times 4 \times 2$ in. will be required for 560 cu. ft. of masonry, allowing $12\frac{1}{2}\%$ for the mortar?

77. The proceeds of a note discounted at a bank for 45 days at 12% without grace amounted to \$682.50; what was the bank discount?

78. The average person inhales 30 cu. in. of air at a breath, and breathes 20 times a minute, and when one tenth of a given bulk of air is breathed the whole is vitiated, or is unfit for breathing. What time would be required for two persons to vitiate the air in a bed-chamber 16×14 ft., and 12 ft. high?

79. Under the conditions given in the preceding problem, how many cubic feet of fresh air per hour should be supplied to a hall $75 \times 56 \times 28$ ft., containing 800 persons, in order that the air in the hall may not become vitiated within 2 hr.?

80. If a bill of goods invoiced at \$750 and bought at a discount of 20% off on 60 days' credit, is sold at an advance of 25% on invoice-price, on 30 days' credit, what per cent is gained, money being worth 6%?

81. Frank pledged his watch to a pawn-broker for \$25; at the end of 40 da. he redeemed his watch for \$26.50. What rate of interest did he pay?

82. A sold a house and lot at a gain of 15%; had he sold it for \$65.40 more he would have gained 18%. For what sum did he sell it?

83. A certain room is 18×20 ft., and 13 ft. high, and has 5 windows and 3 doors. Find the cost of papering both walls and ceiling with paper costing 65¢ per roll, and of carpeting with material 28 in. wide, laid lengthwise of the room, and costing \$1.60 per yard.

84. Maynard & Brown fail, with the following statement of resources and liabilities :—

Resources.		Liabilities.	
Cash,	\$ 786.50	Preferred Claims,	\$ 9650.00
Mdse. as per Invoice,	26495.80	Notes,	24578.60
Collectable Notes,	13548.00	Personal Accounts,	47385.80
Personal Accounts,	12580.95		

After reserving \$1500 to pay expenses of insolvency proceedings, what amount on the dollar can be paid to the general creditors?

85. A bin 26 ft. 8 in. long, 7 ft. 6 in. wide, and 5 ft. 3 in. deep, is filled with potatoes; what are they worth at \$22.50 per ton?

86. Bought coffee at 18¢ per pound and after roasting, sold it at 25¢ per pound. What per cent did I gain, provided the coffee shrunk $12\frac{1}{2}\%$ in roasting?

87.

Dr.		HENRY BROWN & Co.				Cr.
1894.				1894.		
Mar. 6	To Mdse., 60 da.	\$460	Mar. 20	By Cash,	\$250	
June 7	“ “ 30 “	640	June 25	“ “	500	
Aug. 18	“ “ 90 “	850	Sept. 20	“ Draft, 10 da.,	450	

What sum will balance the above account on Oct. 1, 1894, the current rate of interest being 6%?

88. Smith owes Brown on account \$460, due June 1, and Brown owes Smith on account \$350, due May 15, of the same year. On April 10 they settle their accounts, Brown accepting Smith's note without interest for the difference of their accounts. When should this note fall due?

89. If it is 2.35 P. M. local time at Lon. $79^{\circ} 50'$ E., what is the hour at Lon. $23^{\circ} 40'$ W.?

90. A bicycle rider left Jonesville at 4.25 A. M., and arrived at Stapleton, 95 miles away, at 3.10 P. M. If his stoppages amounted to 1 hr. 25 min., what was his average riding-rate per hour?

ANSWERS.

Page 17	Page 19, Cont.	Page 23	Page 28
1. 400330	10. \$17625	12. \$1908	1. $869\frac{2}{3}$
2. 551854	11. \$202	13. \$650	2. $20493\frac{2}{5}$
3. 564873	12. \$20280	14. 24440 cts. or \$244.40	3. $11234\frac{5}{7}$
4. 602151045	Page 20	15. \$1580	4. $43115\frac{1}{6}$
5. 870508	13. \$8024	16. 2352 mi.	5. $45626\frac{4}{8}$
6. 62848546	14. \$35904	Page 25	6. $681487\frac{1}{1}$
Page 18	15. \$163	1. 4028000	7. $22406\frac{4}{3}$
1. \$1480	16. \$3411	2. 16128000	8. $6933368\frac{5}{6}$
2. \$385	17. \$54620	3. 14720000000	9. 365501
3. \$2997	18. \$12516	4. 7580000	10. $252026\frac{4}{3}$
4. \$47111	19. 15037 bu. w. 6156 bu. o. 7350 bu. c. Total, 28543 bu.	5. 1348000	11. $427150\frac{1}{8}$
5. \$7450	Page 22	6. 103224000	12. $5953408\frac{5}{7}$
6. \$10188	1. \$20736	7. 1431000	13. $437881\frac{10}{11}$
7. \$23185	2. \$190508	8. 86724	14. $547603\frac{10}{12}$
8. \$48189	3. \$100072	9. 53946	15. $754062\frac{1}{4}$
9. \$213145	4. \$335016	10. 7629237	16. $519042\frac{4}{9}$
10. \$682438	5. \$13860	11. 8232	17. $403527\frac{37}{50}$
11. \$14440218	6. \$14700	12. 235528	18. $138161\frac{24}{700}$
12. \$6345945	7. \$12145	13. 200385	19. $770\frac{8416}{000}$
Page 19	8. 20475 cts. or \$204.75	14. 10708116	20. $698041\frac{096}{1200}$
1. 3306 mi.	9. \$74880	15. 305592	21. $12104\frac{680}{790}$
2. 12198 bu.	10. \$8432	16. 245672	Page 29
3. \$9750	11. 5472 cts. or \$54.72	17. 1055106	1. $13766\frac{3}{13}$
4. \$3354		18. 269001	2. $15666\frac{4}{19}$
5. \$7435		19. 75828	3. $103249\frac{2}{39}$
6. \$2450		20. 5793648	4. $31306\frac{25}{28}$
7. \$3820		21. 51803874	5. $100002\frac{18}{54}$
8. \$1815			6. $31683\frac{92}{247}$
9. 50 bu.			7. $144591\frac{50}{826}$
			8. $10061\frac{53}{285}$

Page 29, Cont.	Page 31, Cont.	Page 34	Page 38, Cont.
9. 73938 $\frac{286}{327}$	9. 79 $\frac{1755}{17896}$ times	10. 6 lbs.	8. 5, 5, 5, 5, 73
10. 100195 $\frac{463}{601}$	— 2 —	— 2 —	9. 2, 3, 3, 3, 5,
11. 77077 $\frac{3112}{3586}$	1. 4640 $\frac{2}{9}$ acres	1. \$42.09	7, 11, 17
12. 1000797 $\frac{1269}{2761}$	2. \$14.82 $\frac{234}{268}$	2. \$181.65	10. 3, 3, 3, 7,
13. 97501 $\frac{24}{450}$	3. 326 trees	3. \$28.50	11, 257
14. 25856 $\frac{558}{7800}$	4. \$1.27 $\frac{1789}{7860}$	4. Lost \$3310.75	
15. 101001 $\frac{123}{680000}$	Page 32	Page 35	Page 42
16. 14811 $\frac{6560100}{15800000}$	5. 39 $\frac{36}{96}$ ft.	5. 650 lbs.	1. 48
17. 10010201 $\frac{160}{5620000}$	6. \$24.20 $\frac{500}{700}$	6. \$3715.20	2. 72
18. 2546097 $\frac{616}{97640}$	7. 30 $\frac{3}{19}$ ft.	7. \$225	3. 28
19. 20220 $\frac{489020}{989600}$	8. \$3.60 $\frac{55}{97}$	8. \$134.40	4. 52
20. 420 $\frac{319006}{856794}$	9. 86 $\frac{50}{75}$ cts.	9. 15 cts.	5. 37
Page 30	10. 49 $\frac{1480}{1520}$ bu.	10. 261 rds.,	6. 29
1. 6 $\frac{2}{3}$	— 2 —	32 rds.	Page 43
2. 400	1. \$25	11. 1034 mi.	7. 21
3. 144	2. 32 horses	12. 96 mi.	8. 8
4. 12	3. 62 bu.	13. 13 yds.	9. 15
5. 4128 $\frac{48}{73}$	4. 13 times	14. \$91.50	10. 18
6. 195	5. 53 $\frac{10}{35}$ hours	15. 45 p., \$28.65	Page 45
7. 455	6. 54 $\frac{16}{46}$ acres	16. \$106.25	1. 12600
8. 10 $\frac{2}{3}$	7. 2295 $\frac{40}{56}$ bu.	Page 36	2. 1440
9. 34 $\frac{10}{133}$	Page 33	17. \$37.40	3. 10800
10. 892 $\frac{1}{2}$	8. 718 $\frac{14}{87}$ lbs.	18. \$3297.30	4. 8190
— 2 —	9. 22 $\frac{1}{2}$ bu.	19. 805 lbs.	5. 1960
1. 200 cows	10. 446 $\frac{24}{56}$ days	20. 20 men	6. 1440
2. 54 rows, 18 t.	— 2 —	Page 38	7. 14280
3. 6 da., 250 r.	1. 16 lbs.	1. 2, 2, 3, 3	8. 18900
Page 31	2. $\frac{5}{6}$ cts.	2. 2, 2, 2, 2, 3, 3	9. 126000
4. 1400 bu.	3. 38.88 lbs.	3. 2, 2, 2, 2, 5, 7	10. 1260
5. 220 sec.	4. $\frac{7}{8}$ sheet.	4. 2, 2, 3, 7, 11	Page 48
6. 3 m. 40 sec.	5. 8 cts.	5. 2, 3, 3, 19	1. $\frac{5}{7}$
7. 301 gal.	6. 112 sq. in.	6. 3, 5, 83	2. $\frac{1}{2}$
217 cu. in.	7. 14 cts.	7. 2, 5, 5, 17, 41	3. $\frac{4}{11}$
8. 5143 a. 120 rds.	8. 14 cts.		
	9. \$7		

Page 48, Cont.	Page 50, Cont.	Page 52, Cont.	Page 54, Cont.
4. $\frac{41}{81}$ 5. $\frac{3}{40}$ 6. $\frac{3}{4}$ 7. $\frac{1}{3}$ 8. $\frac{85}{432}$ 9. $\frac{4}{5}$ 10. $\frac{3}{5}$	10. $\frac{820}{180}, \frac{170}{180}, \frac{216}{180}, \frac{171}{180}, \frac{540}{180}, \frac{810}{180}$ — 2 — 1. $\frac{7}{2}$ gal. 2. $\frac{9}{4}$ ft. 3. $\frac{16}{8}$ in. 4. $\frac{51}{8}$ mi. 5. $\frac{65}{4}$ ft. 6. $\frac{167}{7}$ cds. 7. $\frac{835}{8}$ mi. 8. $\frac{4227}{11}$ bbl. 9. $\frac{4131}{8}$ bu. 10. $\frac{23693}{6}$ a.	10. $442\frac{1}{34}$ 11. $1324\frac{29}{30}$ 12. $7\frac{377}{1360}$ 13. 245^1 yds. 14. 214^2 yds. 15. 190^9 lbs. 16. 271^{15} lbs. 17. $174\frac{7}{16}$ a. 18. $388\frac{7}{24}$ gross 19. $12\frac{23}{40}$ 20. $106\frac{1}{18}$	2. $\frac{7}{12}$ 3. $\frac{8}{15}$ 4. $\frac{13}{42}$ 5. $\frac{27}{152}$ 6. $\frac{18}{77}$ 7. $\frac{1}{6}$ 8. $\frac{1}{20}$ 9. $\frac{1}{42}$ 10. $\frac{1}{130}$ 11. $\frac{1}{132}$ 12. $\frac{4}{21}$
Page 49	Page 51	Page 54	Page 56
1. $\frac{6}{16}$ yds. 2. $\frac{20}{35}$ lbs. 3. $\frac{9}{27}$ ft. 4. $\frac{30}{144}$ oz. 5. $\frac{27}{3}$ 6. $\frac{540}{15}$ 7. $\frac{252}{9}$ 8. $\frac{32352}{24}$ 9. $\frac{192681}{79}$ 10. $\frac{10322}{13}$	1. $5\frac{1}{2}$ lbs. 2. 29 gal. 3. $3\frac{1}{8}$ yds. 4. $4\frac{1}{8}$ bu. 5. 64 ft. 6. $72\frac{4}{5}$ bbl. 7. $1345\frac{3}{5}$ yds. 8. $42\frac{11}{27}$ hrs. 9. $91\frac{5}{36}$ bbl. 10. $40\frac{13}{25}$ gal.	1. $\frac{1}{4}$ 2. $\frac{2}{9}$ 3. $\frac{1}{2}$ 4. $\frac{4}{15}$ 5. $\frac{13}{35}$ 6. $\frac{1}{6}$ 7. $\frac{13}{24}$ 8. $\frac{17}{50}$ 9. $\frac{1}{6}$ 10. $\frac{3}{16}$ 11. $2\frac{1}{12}$ 12. $2\frac{5}{24}$ 13. $5\frac{1}{6}$ 14. $9\frac{3}{5}$ 15. $567\frac{1}{10}$ 16. $17\frac{1}{8}$ 17. $353\frac{17}{20}$ 18. $323\frac{113}{120}$ 19. 11^2 yds. 20. 33^1 yds. 21. 158^{12} lbs. — 2 — 1. $\frac{5}{6}$	1. $\frac{1}{2}$ 2. $5\frac{1}{4}$ 3. $\frac{5}{18}$ 4. $6\frac{1}{2}$ 5. $\frac{5}{6}$ 6. $4\frac{7}{8}$ 7. $12\frac{2}{15}$ 8. $7\frac{7}{8}$ 9. $\frac{15}{32}$ 10. $1\frac{7}{8}$ 11. $5\frac{5}{32}$ 12. $7\frac{7}{12}$ 13. $10\frac{7}{24}$ 14. $18\frac{7}{16}$ 15. $\frac{1}{4}$ 16. $\frac{1}{20}$ 17. $\frac{5}{144}$ 18. $3\frac{3}{224}$ 19. $\frac{5}{256}$ 20. $1\frac{7}{8}$ — 2 — 1. 1932 2. $343\frac{1}{5}$
Page 50	Page 52		
1. $\frac{9}{12}, \frac{4}{12}, \frac{6}{12}$ 2. $\frac{45}{120}, \frac{80}{120}, \frac{30}{120}, \frac{24}{120}$ 3. $\frac{50}{60}, \frac{35}{60}, \frac{56}{60}$ 4. $\frac{70}{80}, \frac{15}{80}, \frac{68}{80}$ 5. $\frac{224}{280}, \frac{245}{280}, \frac{80}{280}, \frac{140}{280}$ 6. $\frac{210}{315}, \frac{189}{315}, \frac{45}{315}, \frac{70}{315}$ 7. $\frac{16}{60}, \frac{150}{60}, \frac{45}{60}, \frac{50}{60}$ 8. $\frac{108}{12}, \frac{27}{12}, \frac{6}{12}, \frac{44}{12}, \frac{78}{12}$ 9. $\frac{54}{180}, \frac{45}{180}, \frac{130}{180}, \frac{1260}{180}, \frac{100}{180}, \frac{2970}{180}$	1. $1\frac{11}{12}$ 2. $21\frac{27}{360}$ 3. $21\frac{61}{72}$ 4. $7\frac{341}{1260}$ 5. $7\frac{99}{140}$ 6. $44\frac{2993}{3780}$ 7. $312\frac{5}{8}$ 8. $843\frac{9}{70}$ 9. $226\frac{43}{120}$		

Page 56, Cont.	Page 58, Cont.	Page 60	Page 64
3. 2850	10. 688	16. $45\frac{1}{2}$ acres	1. $\frac{1}{12}$
4. $618\frac{3}{8}$	11. $\frac{5}{12}$	17. $21\frac{3}{8}$ yds.	2. $\frac{1}{32}$
5. $6239\frac{1}{16}$	12. $\frac{4}{7}$	18. \$1.75	3. $\frac{1}{22}$
6. 481	13. $179\frac{1}{5}$	19. 174 acres	4. $\frac{2}{59}$
7. $279\frac{5}{6}$	14. $91\frac{1}{8}$	20. $3\frac{3}{7}$ days	5. $\frac{2}{7}$
8. $540\frac{5}{6}$	15. $564\frac{1}{51}$	21. $5\frac{5}{11}$ min.	— 2 —
9. $226\frac{1}{8}$	16. $262\frac{5}{11}$	22. $\$6.05\frac{1}{8}$	1. \$9216
10. $1699\frac{1}{8}$	17. $1155\frac{27}{4}$	23. 60 days	2. \$5100.75 Coll. \$408.06 Com.
Page 57	18. $1854\frac{1}{9}$	24. $22\frac{2}{47}$ days	3. \$453
1. $45\frac{9}{16}$	19. $51\frac{17}{7}$	25. $\frac{3}{32}$, \$6060	4. \$64.05
2. $93\frac{4}{9}$	20. $41\frac{1}{50}$	26. \$50.75	5. $6\frac{1}{8}$ chestnuts
3. $14\frac{4}{19}$	21. $44\frac{8}{19}$	Page 61	6. 658
4. $35\frac{3}{4}$	22. $28\frac{9761}{12968}$	27. 5640 lbs.	7. \$42.75
5. $80\frac{3}{4}$	23. $48\frac{53}{800}$	28. 120000 lbs.	8. $\frac{9}{11}$
6. $93\frac{3}{4}$	24. $144\frac{266}{173}$	29. \$464.75	9. Jas. \$63.45 Jno. \$35.25
7. $109\frac{1}{4}$	Page 59	30. $21\frac{47}{180}$ ft.	10. Carriage, \$98.35 Horse, \$56.20
8. $591\frac{3}{16}$	1. $5\frac{7}{12}$, $1\frac{1}{12}$	31. \$44.43 $\frac{3}{4}$	11. Father \$17.20 Son \$8.60
9. $31\frac{1}{9}$	2. \$2250	32. $28\frac{7}{8}$ lbs.	12. $\$3.93\frac{3}{4}$
10. $155\frac{5}{16}$	3. \$6000 ~	33. \$83.25	Page 62
11. $138\frac{3}{4}$	4. \$675.75	1. \$240.36	1. \$240.36
12. $215\frac{1}{9}$	5. $33\frac{1}{3}$ cts.	2. $1\frac{7}{8}$ ft.	2. $1\frac{7}{8}$ ft.
13. $701\frac{4}{5}$	6. $165\frac{3}{4}$ mi.	3. \$64.35	3. \$64.35
14. $96\frac{1}{16}$	7. \$7.90 +	4. \$4560	4. \$4560
15. $309\frac{2}{3}$	8. 3 bu.	5. \$72.36	5. \$72.36
Page 58	9. $7\frac{7}{18}$ hrs.	6. $\$3\frac{1}{3}$	6. $\$3\frac{1}{3}$
1. $\frac{8}{9}$	10. \$3824 in farm, \$956 in farm stock, Tot. Invest., \$5730	Page 63	1. \$228.95
2. $1\frac{1}{3}$	11. 24 lots	1. \$228.95	2. $8\frac{1}{4}$
3. $1\frac{1}{4}$	12. $264\frac{1}{2}$ yds.	3. $10\frac{1}{2}$	3. $10\frac{1}{2}$
4. $5\frac{1}{3}$	13. $9\frac{1}{7}$ cts.	4. \$141	4. \$141
5. $\frac{3}{2}$	14. \$25.32	5. 7	5. 7
6. $\frac{7}{10}$	15. Tot. Amt. \$64000 \$8000 in farm prop. \$28000 in city prop. \$14000 in stocks and bonds	6. $19\frac{1}{4}$	6. $19\frac{1}{4}$
7. $5\frac{3}{5}$			
8. $\frac{2}{5}$			
9. $\frac{2}{2}$			
			Page 65
			13. 24 bu.
			14. \$842.10
			15. A $\frac{1}{8}$, B $\frac{1}{2}$, C $\frac{1}{6}$
			Page 67
			1. .5
			2. .17
			3. .07
			4. .124
			5. .003
			6. .304
			7. .0564
			8. .0456
			9. .00498

Page 67, Cont.	Page 69, Cont.	Page 71, Cont.	Page 73, Cont.
10. .03648	8. $\frac{27}{40}$	4. 364.885775	13. 22.094375
11. .000395	9. $\frac{3}{50}$	5. 20.1275 cords	14. 3.39003
12. 16.142	10. $14\frac{3}{8}$	6. 1933.73 acres	Page 76
13. 20.08	11. $\frac{1}{125}$	Page 72	1. .34
14. 42.0204	12. $14\frac{11}{32}$	7. 2047.54375 miles	2. .005
15. 100.036418	13. $14\frac{11}{32}$	8. $26.286\frac{3}{6}$	3. 21800
16. 16.164	14. $16\frac{51}{200}$	9. 28.654	4. 2.5
Page 68	15. $\frac{235491}{250000}$	10. .6752	5. .0000125
1. .125	16. $\frac{1087}{10000}$	11. $3.309\frac{5}{6}$	6. 690000
2. .8	Page 70	12. $2.479\frac{3}{4}$	7. 30.3
3. .75	1. (1) .857142	Page 73	8. 1389.4736+
4. .16	(2) .5	1. 4.208	9. 130
5. .05078125	(3) .36	2. 6.213	10. 106500
6. .46875	(4) .916	3. $10.000\frac{8}{9}$	11. $3428.5714+$
7. .875	(5) .285714	4. 9.6023	12. 50
8. .12	2. (1) $.54545\frac{5}{11}$	5. 24.033	13. 22
9. .1875	(2) $.38461\frac{7}{3}$	6. 21.033	14. 3.025
10. .05	(3) $.14285\frac{5}{7}$	7. $2.999\frac{1}{4}$	15. .35
— 2 —	(4) $.58333\frac{1}{3}$	8. 1.86	16. 20.1
1. 11.25	(5) $.38888\frac{8}{9}$	9. .297	17. 32406.25
2. 17.125	3. (1) $.101428\frac{4}{7}$	10. 9.9984	18. 163020
3. 14.1875	(2) .241875	— 2 —	19. 11270.1101
4. 15.5	(3) $.012071\frac{3}{7}$	1. 18.128	20. 10001100.16211
5. 16.6	(4) $.0617641\frac{2}{7}$	2. 14.812	— 2 —
6. 15.025	(5) $.571428\frac{4}{7}$	3. .07443844	1. .95
7. 14.03125	4. (2) $\frac{73}{90}$	4. .0016605	2. $\frac{5}{8}, \frac{3}{50}$
Page 69	(3) $\frac{19}{30}$	5. 1.02592	3. .0001
1. $\frac{5}{8}$	(4) $\frac{61}{800}$	6. .225225	4. .65
2. $\frac{3}{40}$	(5) $\frac{17}{30}$	7. 7.744	5. 284125.7 lbs.
3. $5\frac{1}{50}$	(6) $\frac{13}{150}$	8. .00000504	6. \$21.11+
4. $15\frac{3}{80}$	Page 71	9. 278.5	7. 243 yds.
5. $12\frac{53}{800}$	1. 67.3348	10. 216	8. \$446.40
6. $\frac{37}{40}$	2. 1551.1093	11. 9648.1	Page 77
7. $24\frac{11}{20}$	3. 10651.49835	12. .058625	9. 18.5914+ bbl.
			10. .25

Page 77, Cont.	Page 83	Page 84, Cont.	Page 87, Cont.
11. $742\frac{4}{13}$ cu. yd.	1. \$630	6. \$267.33	7. 520 lbs.
12. \$93.90	2. \$1500	7. \$1.42	8. 43 lbs.
13. \$931.50	3. \$755	8. \$14.82	9. 41.76 or $41\frac{1}{2}$ lbs.
14. 1740 lbs. coffee, 870 lbs. tea, 15818.1818+ or 15818 $\frac{2}{11}$ lbs. sugar, 11136 lbs. spices, 1242 $\frac{2}{3}$ bu. potatoes	4. \$362.50 5. \$150 6. \$225 7. \$273 8. \$265 9. \$299.20	9. \$1194.20 10. \$17.78	10. 544 lbs. 11. 173 lbs. 12. 2788 lbs.
15. \$560.19+	10. \$500	Page 85	BILLS
16. 39.8009+ hrs.	11. \$65.25	1. \$5.46	Page 90
17. \$38573.32+	12. \$937.50	2. \$27.05	Bal., \$12.09
18. \$1621.03+	— 2 —	3. \$121.25	Page 91
19. 1545.7 bu.	1. \$19.02	4. \$803.04	Amt., \$34.27
20. \$4255.50 in city property, \$709.25 in farm prop., \$567.40 in mortgages, \$141.85 in grain; 315.2+ or $315\frac{2}{3}$ bu.	2. \$5.70 3. \$280.80 4. \$91.97 5. \$66.12 6. \$113.47 7. \$162 8. \$364.05 9. \$108.54	5. \$30.82 6. \$79.26 7. \$88.68 8. \$59.62 9. \$68.25 10. \$135.33	Page 92 \$79.95
Page 82	Page 84	Page 86	Page 93
1. \$158	10. \$33	1. \$15.05	Amt., \$8.85
2. \$198	11. \$237	2. \$30.34	Page 94
3. \$46	12. \$249.84	3. \$255.33	Bal., \$20.88
4. \$45	13. \$139.62	4. \$6592.04	Page 95
5. \$197	14. \$633.79	5. \$1911.70	1. \$3057.73
6. \$382.25	15. \$668.12	6. \$7179.94	2. \$227.38
7. \$416	— 2 —	Page 87	3. \$498.27
8. \$323	1. \$3.14	7. \$6188.75	Page 96
9. \$141	2. \$13.84	8. \$2881.55	4. \$241.70
10. \$119	3. \$47.40	— 2 —	5. \$987.32
11. \$104	4. \$13.40	1. 1100 lbs.	6. \$13.81
12. \$208	5. \$20.26	2. 496 lbs.	7. \$385.42
13. \$4.84		3. 5544 lbs.	Page 97
14. \$48		4. 2268 lbs.	8. \$233.84
15. \$99.06		5. 420 lbs.	9. \$7.50
16. \$119.08		6. $50\frac{4}{5}$ lbs.	10. \$370.40

Page 106	Page 107, Cont.	Page 109, Cont	Page 112, Cont.	
1. 250 mills	12. 5 da. 23 hrs. 28 min. 9 sec.	2. .8125 bu.	2. 8 yr. 4 mo. 15 da.	
2. 25150 mills	13. 20 cn. 28 ft.	3. .8181+ rds.	3. 2 yr. 3 mo. 7 da.	
3. 255 s.	14. 3sq.yd. 8sq. ft. 124 sq. in.	Page 110		
4. 24208 far.	15. 51 gal. 2 qt.	4. .46875 mi.	4. 2 yr. 3 mo. 28 da.	
5. 2325 pwt.	Page 108		5. 40 da.	
6. 136432 oz.	1. 5s.	5. .875 cd.	6. 113 da.	
7. 547200 grs.	2. 8 oz.	6. .14 ton	7. 48 da.	
8. 3764 yds.	3. 1500 lbs. or 15 cwt.	7. £.775	8. 208 da.	
9. 1166 in.	4. 20 hrs.	8. .52083+ da.	Page 113	
10. 16300 links	5. 7 qt.	9. .71875 lb.	9. 163 da.	10. 232 da.
11. 67 rds.	6. 72 sq. rd. 22 sq. yd.	10. .4444+ cu. yd.	11. 128 da.	
Page 107		— 2 —	12. 193 da.	— 2 —
12. 9075 sq. yd.	7. 1 pk. $1\frac{1}{5}$ pt.	1. 46 bu. 1 pk. 5 qt. 1 pt.	1. £82 13s. 4d.	2. 60 gal. 3 qt. 2 gills
13. 32801 sq. in.	8. 60 rd.	2. 64 T. 10 cwt. 55 lb. 5 oz.	3. 270 da. 5 hr. 4 m.	3. 270 da. 5 hr. 4 m.
14. 150336 cu. in.	9. 2 qt. $3\frac{1}{5}$ gills	3. £167 18s. 2d.	4. 178 lb. 7 oz. 4 pwt.	4. 178 lb. 7 oz. 4 pwt.
15. 1024 cu. ft.	10. 23 cu. ft. 1080 cu. in.	4. 12 T. 1395 lb.	5. 51 mi. 201 rd. 1 yd. 6 in.	5. 51 mi. 201 rd. 1 yd. 6 in.
16. 122 pts.	11. $\frac{5}{252}$ yd.	Page 111		6. 112 sq. rd. 10 sq. yd. 2 sq. ft.
17. 480 qts.	12. $\frac{7}{1280}$ a.	5. 146 gal. 1 qt.	7. 796 gal. 3 qt. 1 pt.	7. 796 gal. 3 qt. 1 pt.
18. 12000 sheets	13. $\frac{5}{96}$ lb.	6. 79 mi. 159 rd. 4 yd. 8 in.	8. 395 mi. 180 rd. 2 yd.	8. 395 mi. 180 rd. 2 yd.
19. 2160 articles	14. $\frac{7}{160}$	— 2 —	9. 513 lb., \$112.86	9. 513 lb., \$112.86
20. \$33.58	15. $\frac{11}{324}$ cu. yd.	1. 5 lb. 10 oz. 1 pwt.	10. 58 rd. 1 yd. 2 ft. 6 in.	10. 58 rd. 1 yd. 2 ft. 6 in.
— 2 —	16. $\frac{3}{56}$ lb.	Page 114		1. 2 hr. 54 min. 48 sec.
1. 1 mi. 54 rd. 1 yd.	Page 109		2. 13 cwt. 86 lb.	
2. 84 bu. 2 pks. 6 qt.	1. 1 ft. 1.5 in.	2. 2 mi. 65 rd. 2 ft.	3. 9 hr. 34 min 5 sec.	3. 9 hr. 34 min 5 sec.
3. \$5.684	2. 8.82 in.	3. 8 cu. yd. 25 cu. ft. 99 cu. in.	4. 4 cu. yd. 22 cu. ft. 691 $\frac{1}{5}$ cu. in.	4. 4 cu. yd. 22 cu. ft. 691 $\frac{1}{5}$ cu. in.
4. 100 yd. 1 ft. 2 in.	3. 3 pk. 1 qt. 24 pt.	4. 10a. 34sq.rd. 5 sq. yd. 2 sq. ft.	Page 112	
5. 24 cd. 92 cu. ft. or 24 cd. 5 cd. ft. 12 cu. ft.	4. 280 rd.	5. 16 gal. 3 qt. 1 pt.	Page 112	
6. 128 cd.	5. 7cwt. 22 lb. 8 oz.	6. 59 da. 12 hr.	Page 112	
7. £40 4s.	6. 24 gal. 2 qt. 1 pt. 2.272 gi.	Page 112		
8. 1 a. 35 sq. rd. 8 sq. yd. 3 sq. ft. 36 sq. in.	7. 21 min. 23.4 sec.	Page 112		
9. 1 T. 71 lbs. 12 oz.	8. 15.8 oz.	Page 112		
10. 8 lbs. Avoir.	— 2 —	Page 112		
11. 197 reams 5 qr. 1 sheet	1. .625 yr.	Page 112		

Page 114, Cont.	Page 118, Cont	Page 124, Cont.	Page 129, Cont.
5. 5 bu. 2 pk. $1\frac{1}{2}$ qt.	4. \$30.28	5. $96\frac{1}{4}$ acres	9. 10 ft. 7 in.
6. 3 sq. rd. 26 sq. yd. 7 sq. ft. 11 sq. in.	5. \$24.64	6. \$2.35	10. $1178571\frac{2}{7}$ gal.
7. 15 gal. 3 qt. $1\frac{5}{11}$ pt.	6. \$376.65	7. $101\frac{9}{11}$ rds.	11. 5 ft. $2\frac{1}{2}$ in.
8. 9 oz. 11 pwt. $5\frac{5}{7}$ gr.	7. \$2.36	8. 1 min. $42\frac{5}{7}$ sec.	12. 14 ft. $10\frac{2}{11}$ in.
9. 55 lb. $12\frac{2}{3}$ oz.	Page 119	9. $2290\frac{10}{11}$	Page 130
10. 20 sq. rd. 20 sq. yd. 1 sq. ft. 72 sq. in.	1. \$19.35	10. 473 sq. ft.	1. \$9187.50
Page 115	2. \$9.40	Page 125	2. \$4097.22
1. $7\frac{221}{259}$	Page 120	1. $36\frac{1}{5}$ cu. yd.	3. \$6289.06
2. $891\frac{33}{52}$	1. \$8960	2. 66 cu. ft.	Page 132
3. $6\frac{86}{119}$	2. 106880 a.	3. 1647 cu. ft.	1. 36875 lb.
4. $97\frac{133}{411}$	Page 121	4. $98\frac{23}{144}$ cu. yd.	2. 676.5 lb.
Page 116	3. 949.376 a.	5. $102\frac{2}{3}$ cu. ft.	3. 11.3+ pressure per in., 2006875 lb. tot. pressure
1. 360 sq. ft.	4. \$21379.30	Page 126	4. 26.56+ lbs.
2. $332\frac{2}{5}$ sq. yd.	5. \$1032	1. \$69.90	Page 134
4. 120 sq. ft.	6. \$420.57 gain	2. \$53.86	1. 3 hrs. 34 min. 12 sec.
6. $21\frac{2}{3}$ sq. yd.	— 2 —	Page 127	2. 5 min. 48 sec. past 11 A. M.
Page 117	1. 60 rds.	3. 3456 bd. ft.	3. 49 min. 12 sec. past 10 P. M.
7. 36.71+ a.	2. 72 ft.	4. \$48.27	4. 10 min. 16 sec. past 7 P. M.
8. $141\frac{1}{3}$ sq. yd.	3. 1600 rds.	5. \$155.98	5. 3 hrs. 26 min. 40 sec. fast
9. $87\frac{26}{27}$ sq. yd.	4. 26 ch. 60+ l.	— 2 —	— 2 —
10. $89\frac{7}{9}$ sq. ft.	Page 122	1. \$336	1. $19^{\circ} 18'$
Page 118	1. $21\frac{3}{32}$ acres	2. \$1010.10	2. $61^{\circ} 15'$
1. \$75.83	2. \$2.13	3. \$1854	3. $36^{\circ} 15'$ west
2. $46\frac{1}{2}$ yds.	Page 123	Page 129	
3. \$138.75	3. \$5 33	1. 37.8 bu.	
	Page 124	2. 128 bu.	
	1. $201\frac{1}{7}$ sq. rd.	3. $177\frac{13}{20}$ bu.	
	2. 396 yds.	4. 2700 gal.	
	3. $23\frac{43}{56}$ sq. ft.	5. \$82.37	
	4. 326.6 sq. ft.	6. $5598\frac{3}{14}$ gal.	
		7. $46\frac{113}{147}$ bbl.	
		8. $7\frac{7}{9}$ ft.	

Page 135	Page 139, Cont.	Page 141, Cont.	Page 144, Cont.
1. 1:45 P. M.	9. \$5.04	5. 24%	5. \$278.77
2. 3:20 P. M.	10. \$937.50	6. 36%	6. \$313.66
4. 9 min. faster	11. \$1875	7. $12\frac{1}{2}\%$	7. \$301.41
5. 12:10 P. M.	12. 64 a. wheat, 85 $\frac{1}{3}$ a. oats, 170 $\frac{2}{3}$ a. rem.	8. $62\frac{1}{2}\%$ — 2 —	8. \$361.21
Page 136	13. \$250 board, \$5 washing, \$1 charity, \$200 clothes, \$100 other exp. \$444 rem.	1. $1\frac{1}{3}\%$	9. \$317.48
1. \$410.15	14. \$34.75	2. \$4056000	10. \$266.36
2. 4 da. 6 hrs. 50 min.	Page 140	3. \$1300	11. \$324.90
3. 16 $\frac{3}{4}$ tons	1. \$820	4. 680	Page 145
4. \$617.76	2. \$500	5. A 40%, B 60%	12. \$2.39
5. 3136 lb.	3. \$415	Page 142	13. \$529.86
6. \$964.38	4. \$53	6. \$625	14. \$23.33
7. 9 da. 3 hr. 47 min. 40 sec.	5. \$99	7. $64\frac{5}{8}\%$	15. B's offer, 75¢
Page 137	6. \$364.80	8. $33\frac{1}{3}\%$	16. 20%
8. 118+ bbl.	7. 200 acres	9. \$929.65	17. \$117.60
9. \$229.50	8. \$9840 whole, \$6150 rem.	10. 43.7%	18. \$132.30
10. 50 $\frac{19}{11}$ acres	9. \$33600 cap., \$5040 lot, \$10080 build., \$15120 stock	11. \$200	19. \$92.44 cash, \$97.88, 20 da., \$103.31, 45 da., \$108.75, 90 da.
11. \$2290.58	10. \$648	12. \$4950	20. \$156.52
12. \$270.70	11. \$64000	13. \$12.94	21. \$625
13. 73° 27' 18" W.	Page 141	14. 25%, 35%, 40% \$9.69, \$13.57, \$15.50	22. \$34.29
14. \$9672	1. 20%, 30%, 80%	15. \$2000	Page 148
15. 446.4+ bbl.	2. $74\frac{1}{2}\%$, $35\frac{5}{8}\%$	16. 220000 sheep	1. \$42
16. \$13.15	3. 55%	17. 1.85%	2. \$980
Page 139	4. 6%	Page 143	3. \$45
1. \$12.40	Page 144	18. \$10000	4. \$128.50
2. \$9.45	1. \$216	19. $83\frac{1}{3}\%$	5. \$379.50
3. \$43.60	2. \$319.20	20. 1900%	6. \$92.50
4. \$244.40	3. \$568.15		7. \$61.10
5. \$336.21	4. \$628.80		8. \$10.45
6. \$333.33			9. \$1.08
7. \$78.39			10. \$3658
8. \$532			11. \$18.75

Page 148, Cont.	Page 150, Cont.	Page 156, Cont.	Page 158, Cont.
12. 25%	41. Gain \$105,	17. \$26.72	5. \$5.25
13. \$75	Sales \$905	18. \$19.02	6. \$6.13
14. 20%	42. 60% gain,	— 2 —	7. 75 cts.
15. \$5.92	33 $\frac{1}{3}$ % loss	1. \$27.60	8. 67 cts.
16. 20%	43. \$15 net gain,	2. \$29.05	9. 13 cts.
17. 5%	\$202.50 sales	3. \$154.33	10. 27 cts.
18. 38 $\frac{3}{8}$ %	44. \$1295	4. \$373.25	11. \$7.50
19. \$220	45. \$60	5. \$215	12. \$45
Page 149	Page 151	Page 157	13. \$1.17
20. \$120	46. \$31.60 net gain	6. \$25.69	14. 35 cts.
21. 20%	\$485.60 am't. rec'd.	7. \$13.22	15. \$5.85
22. \$1.60	47. \$1012.50	8. \$106.35	16. \$9
23. 116 $\frac{8}{17}$ %	assessed value,	9. \$70.28	17. \$29.02
24. \$8304	44 $\frac{4}{9}$ % gained	10. \$88.97	18. \$31.76
25. \$440	48. \$691.55 sales	11. \$108.31	19. \$146.58
26. 12 $\frac{1}{2}$ %	\$50.12 gain	12. \$68.14	Page 159
27. 4%	49. \$37.50 loss	13. \$80.01	20. \$64.78
28. \$350	Page 156	14. \$128.27	Page 160
29. \$924	1. \$48	15. \$706.88	1. \$500
30. 24 $\frac{4}{9}$ cts.	2. \$177.60	16. \$3.16	2. \$456
31. \$500, \$333.33	3. \$51.04	17. \$5.97	3. \$4368
32. \$265	4. \$234.99	18. \$181.20	4. \$547.50
33. Cost \$3550,	5. \$153.60	19. \$179.52	5. \$1000
S. P. \$4082.50	6. 27 cts.	20. \$645	6. \$500
34. \$1.25	7. \$91	21. \$778.65	7. \$145.84
Page 150	8. \$245.42	22. \$34.40	8. \$768.75
35. \$4350.44	9. \$60.92	23. \$83.50	Page 161
36. 41 $\frac{3}{17}$ %	10. \$9.08	24. \$2850	1. 3 yr. 4 mo.
37. 16 $\frac{2}{3}$ %	11. \$28.32	25. \$1151.50	2. 6 mo.
38. \$208.34	12. \$2.77	Page 158	3. 4 mo. 18 da.
39. \$107.50	13. \$78.75	1. \$14.50	4. 2 yr. 4 mo.
40. 81 $\frac{9}{11}$ % and	14. \$13.44	2. \$1.97	5. Nov. 2, 1892
161 $\frac{9}{11}$ %	15. \$12.05	3. \$1.88	6. 12 yr. 6 mo.
	16. \$221.68	4. \$2.25	

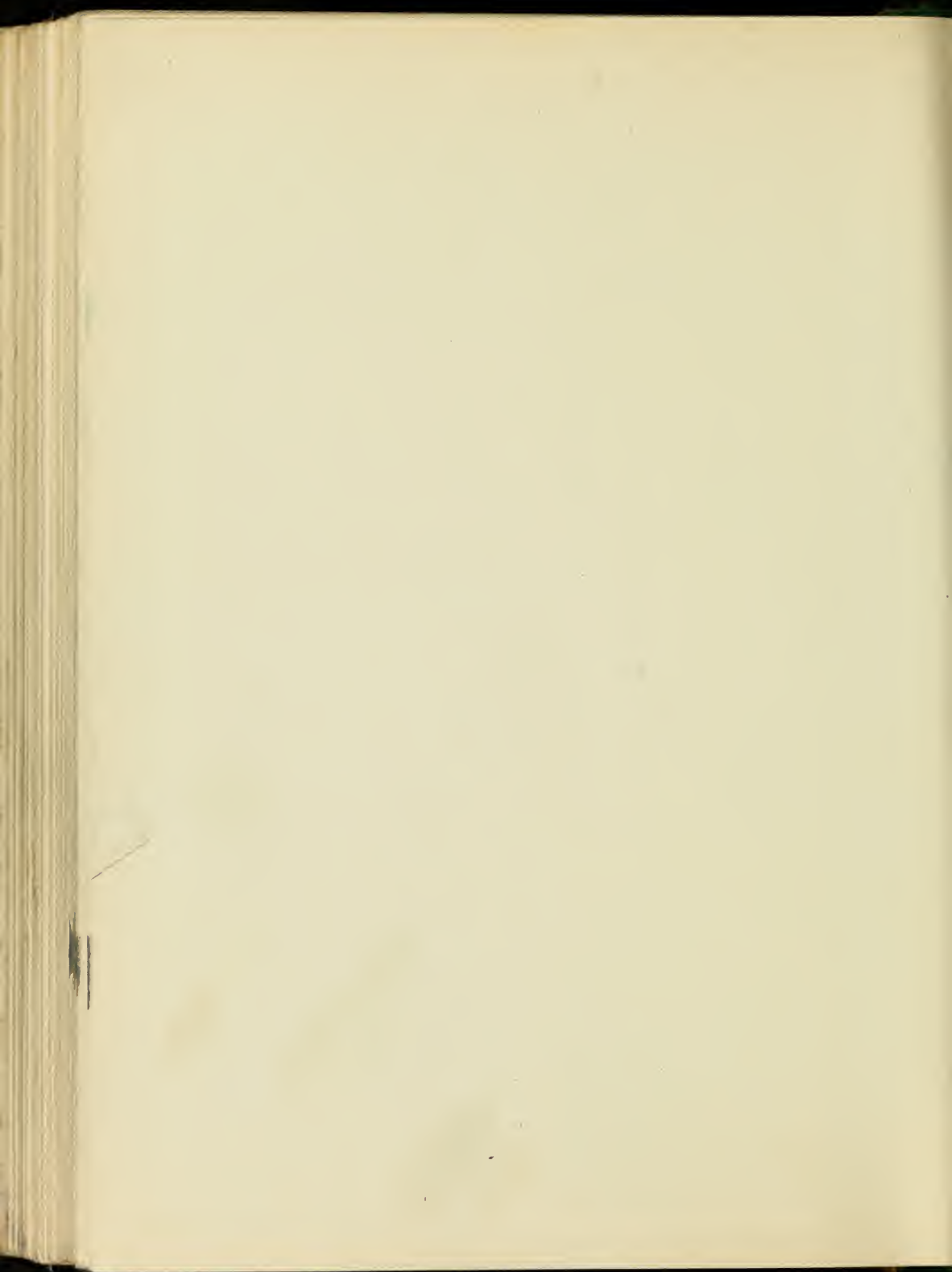
Page 161, Cont.	Page 165	Page 176	Page 180, Cont.
7. 8 mo. — 2 —	1. \$646.52	1. \$171.58	2. a. Sept. 8, '90
1. 8%	2. \$611.25	2. \$119.98	b. 18 da.
2. 6%	3. \$424.65	3. \$609.13	c. \$1.36
Page 162	4. \$407.57	4. \$657.85	d. \$338.64
3. 7%	5. \$27.05	5. \$542.53	3. a. May 4, '92
4. 9%	6. \$1210.39	Page 177	b. 40 da.
5. 8+%	7. \$487.29	6. \$440.63	c. \$6.67
6. 6%	Page 168	7. \$117.89	d. \$1193.33
7. 5%	1. \$527.71	8. \$1051.86	4. a. Oct. 26, '93
— 2 —	2. \$432.22	Page 178	b. 41 da.
1. \$666	3. \$10667.02	1. \$300, \$45	c. \$32.80
2. \$1000	4. \$9879.24	2. \$550, \$55	d. \$3567.20
3. \$291.85	5. \$1712.71	3. \$360, \$4.80	5. a. Apr. 5, '95
4. \$550	Page 169	4. \$500, \$7.75	b. 80 da.
5. \$450	6. \$23736.92	5. \$340, \$5.61	c. \$5.41
6. \$4000	— 2 —	6. 1st. offer, 21¢	d. \$342.59
Page 163	1. \$1200	7. \$1025	6. a. Nov. 3, '94
1. 38 cts.	2. \$500	8. \$5.70 gain	b. 21 da.
2. \$2.21	3. \$1000	9. \$99.03 loss	c. \$5.25
3. \$1.56	Page 173	10. \$473.33	d. \$1494.75
4. \$7.84	1. \$305.62	11. \$29.41 gain	7. \$13.75
5. \$5.30	Page 174	Page 179	8. Dis. 34 cts.
6. \$7.74	2. \$267.99	12. \$176 loss	Pro. \$339.66
7. \$42.31	3. \$162.19	13. \$3564.41	9. Dis. \$3.92
8. \$120.71	4. \$2507.79	14. Cash offer,	Pro. \$836.08
9. \$88.77	5. \$5317.44	\$8.08	10. \$566.38
10. \$178.03	Page 175	Page 180	Page 181
Page 164	6. \$1026.43	1. a. Dec. 14, '91	11. a. Apr. 27, '95
1. \$271.44		b. 60 da.	b. 62 da.
2. \$7656.61		c. \$9.40	c. \$3605.43
3. \$2808		d. \$930.60	12. \$2.54
4. \$2110.62			13. a. 128 da.
			b. \$379
			14. \$2590.99

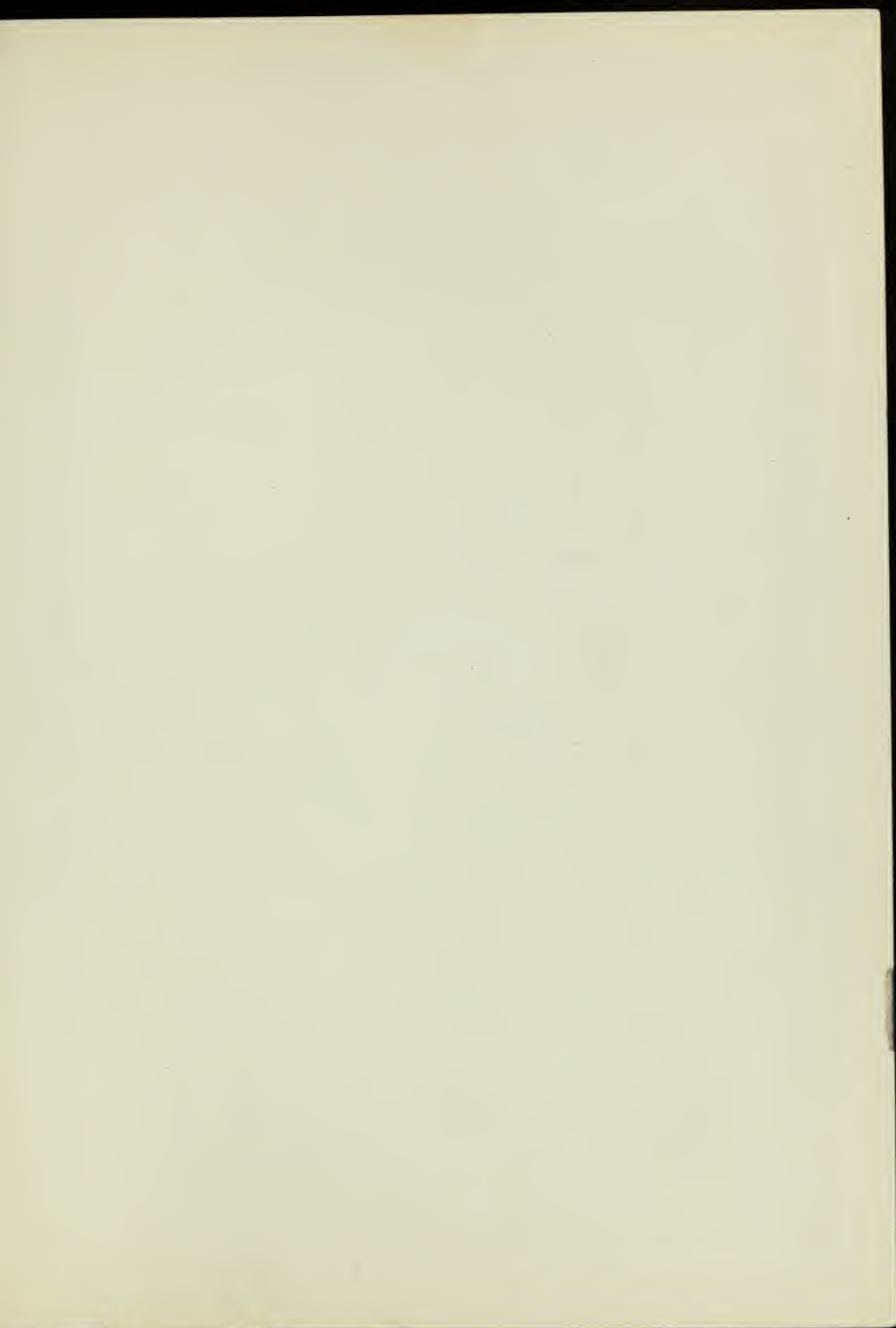
Page 182	Page 186	Page 192	Page 198
1. \$540	22. \$2520	1. \$74.88	2. \$236.25
2. \$923.87	24. \$650	2. \$17	4. Home Co., \$11904.76
3. \$1007.39	25. \$6500	3. \$153	Rochester Co., \$8095.24
4. \$2031.49	26. 3%	4. \$102	5. Queen, \$1777.78
5. \$502.93	27. \$9864.54	Page 193	Mechanics', \$1333.33
6. \$230.35	28. 7%		Globe, \$888.89
Page 184	29. \$4.99		6. \$120
	30. Com. \$618.11		8. 1¼%
	Invest. in groc., \$6661.89		9. \$335.16
	31. \$1300.51	Page 199	
	Page 187		6. \$1.17
7. ½%			
8. 15 mills			
9. \$7500			
10. \$1.10			
12. 2%			
13. \$245.25			
14. 14 mills			
15. Rate 12.3 mills Jones' tax \$215.99 Smith's assm't \$6699.19			
Page 195	Page 190	1. \$325.57	
		2. \$100.44 (See Art. 206)	
		3. \$43.24	
		4. \$237.62	
		5. \$2126.35	
6. \$1155.61			
7. \$843.75			
8. \$18			
9. \$213.75			
10. \$62.40			
11. \$576			
12. \$1210.07			
13. \$1618.76			
14. \$23275.08 tot. duty 42.8 cts. gr. cost per lb.			
Page 203	Page 191	1. Mar. 22, '94	
		2. Nov. 1, '93	
		3. Aug. 25, '94	
		4. Oct. 20, '94	
		5. Nov. 20, '92	
6. Sept. 21, '94			
Page 201	Page 189	7. Sept. 14, '94	
		8. Mar. 21, '95	
		Page 200	
Page 185	Page 188	9. 5%	
		10. 6½%	
		11. 4%	
		12. 1¼%	
		13. 2½% com. 1¼% guar.	
14. \$255			
15. \$2675			
16. \$560			
17. \$3245.50			
18. \$156			
20. \$500			
21. 1500 bu.			

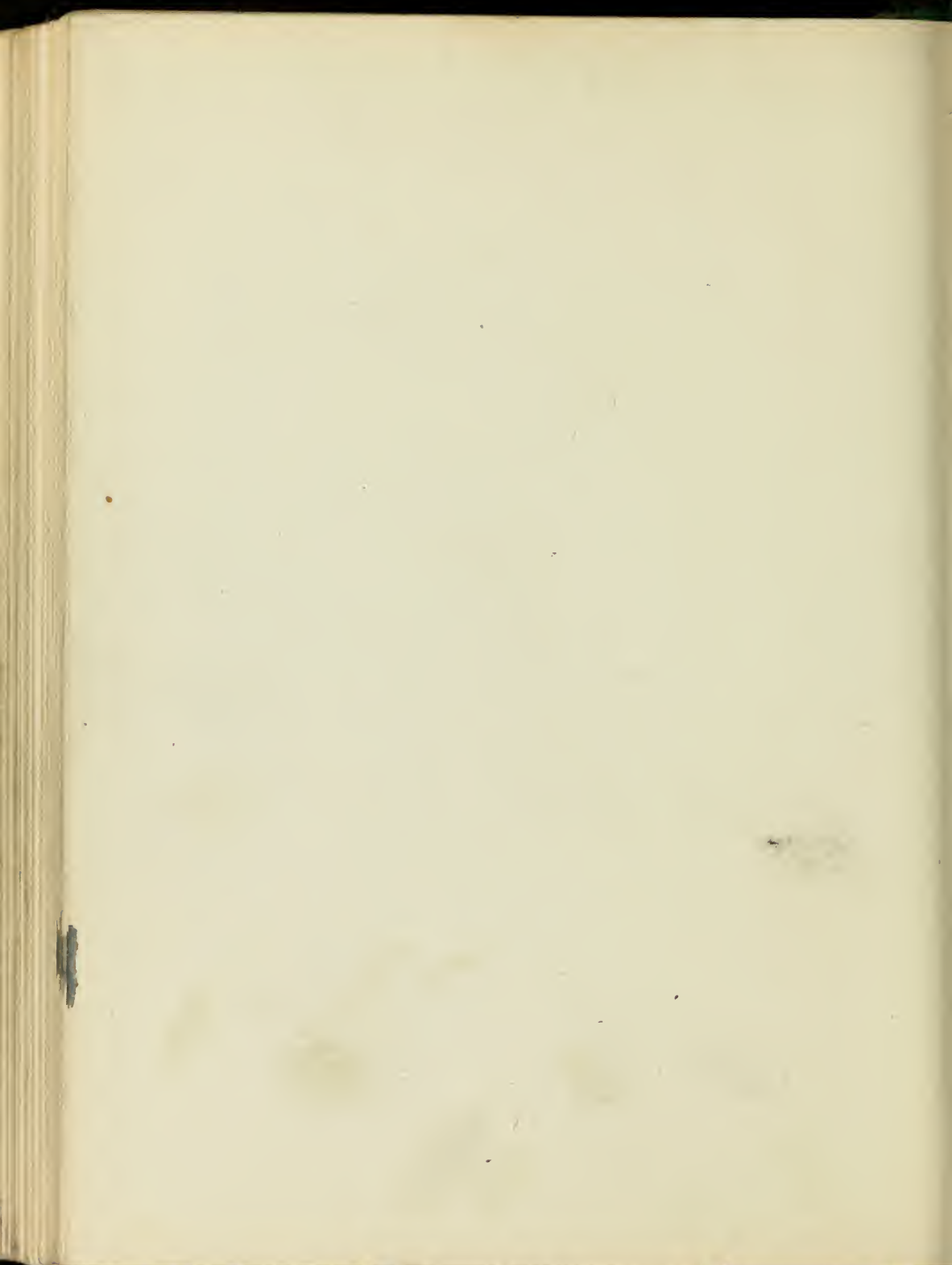
Page 204	Page 214, Cont.	Page 219, Cont.	Page 223, Cont.
7. Apr. 17, '93	— 2 —	3. \$9.60,	2. A. \$1500,
8. Sept. 10, '94	1. \$592.04	\$7.20, \$12	B. \$2400,
9. May. 19, '94	2. \$14757.81	4. \$37.01,	C. \$3600
10. Mar. 2, '93	3. \$4757.94	\$45.83, \$44.06	3. A. \$7000,
	4. \$6110.61	5. \$234.38,	B. \$5500
Page 207	Page 215	\$208.33,	4. \$3900 gain,
1. June 5, '92	5. $99\frac{3}{4}\%$	\$494.79,	J. \$2038.64
2. Nov. 22, '91	Page 217	\$312.50	H. \$1861.36
3. July 24, '93	1. 9	Page 220	5. A. \$1600,
4. May 31, '94	2. 51	1. 270 rd.	B. \$1400
Page 208	3. 33	2. \$350	Page 224
5. Jan. 15, '94	4. 994	3. 9%	6. Pal. \$1705.26
6. \$408.10	Page 218	4. \$540	Pierce \$1894.74
7. \$749.85	1. \$15.20	5. \$1360.80	Page 225
Page 209	2. 99 cts.	Page 221	1. A. \$1350,
8. \$2288.09	3. \$2960	6. $4\frac{4}{5}$ ft.	B. \$1200
9. \$1878.65	4. \$17.50	7. \$42	2. M. \$2668.27
Page 211	5. \$924	Page 222	W. \$2331.73
1. Apr. 21	6. $7\frac{1}{5}$ ft.	1. \$20235	3. A. \$2758.35
Page 213	7. \$1750	2. \$11841	B. \$2123,
1. \$275.49	8. $115\frac{1}{5}$	3. \$2384	C. \$1518.65
2. \$289	9. \$214.20	4. \$944	4. Anson
3. \$177.91	10. 50 men	5. \$11898	\$1156.40,
4. \$239.03	11. \$1.04	6. \$12750.25	Ander. \$1093.60
5. \$1100	12. 32 da.	7. \$14200	Page 226
Page 214	Page 219	8. \$12850,	5. A. \$2809.06,
7. \$747.75	13. $11\frac{1}{5}$ cts.	\$1550	B. \$2790.94
8. \$9505.97	14. 20 da.	9. \$3298	Page 227
9. \$1478.50	— 2 —	Page 223	1. 75 cts.
	1. \$35.88,	1. K. \$2250,	2. $76\frac{9}{16}\%$,
	\$53.82	B. \$1650	\$2450
	2. John \$13.50,		3. $71\frac{46}{89}\%$
	James \$16.20		

Page 227, Cont.	Page 230	Page 232, Cont.	Page 234, Cont.
4. $70\frac{355}{88}\%$, A. \$9398.25, B. \$5410.69, C. \$7341.06	19. \$4.75 20. \$11.45 21. \$15 22. Cash \$296.12 20 da. \$312.57, 45 da. \$322.44, 90 da. \$329.02	42. 8:55 P. M. 43. 66 ft. 44. 38— cts. 45. \$3681.84 46. 10 mi. per hr. 47. 2.29166 48. 2.29164 49. 3565 lb. 50. \$221.59 51. \$204.50	73. 80 cts. Page 235 74. \$453.31 75. \$240 loss 76. 13230 bricks 77. \$10.39 78. 6 hr. 27 min. 79. 10786.6 cu. ft. 80. 57% 81. 54% 82. 2507 83. Paper \$23.40 Carpet \$85.33
Page 228	23. \$4671 24. \$300 25. 20 ft. 26. \$191.10	Page 233	Page 236
7. 62% \$2157.60 — 2 —	Page 231	52. \$16.28+ 53. $17\frac{11}{17}\%$ 54. \$768 55. 133.6 bu. 56. \$184.29 57. \$1753 58. \$246.80 59. 30.7% 60. \$864.13 61. \$30.40 62. \$461.70	84. $58.7+\text{¢}$ per \$1 85. \$446.51 86. $21.5+\%$ 87. \$748 88. July 25 89. 7:41 A. M. 90. $10\frac{5}{8}$ mi.
1. \$1278 2. 38 yds., \$28.45 3. \$22228.70 4. 13.4 bu. 5. 18 ft. 6. \$2553	27. \$73837.50 Value of part remaining, .18602 part sold 28. \$365.48 whole gain, $8.94+\%$ gained 29. $25\frac{5}{19}\%$ 30. \$1600 31. $55714\frac{2}{7}$ 32. \$81 dimin. 33. 14+ cts. 34. \$267.26 35. $43\frac{5}{16}$ tons 36. \$3141.88+ 37. 6000 lbs.	Page 234	
Page 229	Page 232	63. \$900 64. $3685\frac{5}{7}$ lb. 65. 65.1 lb. 66. \$180.99 67. \$1615 68. \$294.55 69. \$152 70. \$562.59 71. 895.62 72. \$6207.50	
7. $30.5+^{\circ}$ 8. \$1.05 9. 2533 10. \$11 11. \$83.95 12. \$4500 13. 16 yds., \$12.75 cost of making 14. 29 ft. 15. $14\frac{9}{16}$ in. 16. $31\frac{23}{96}$ sq. ft. 17. \$2.76 18. $59\frac{3}{8}$ acres	38. 170 da. 39. \$3.50 40. \$19.53+ 41. \$8076.25		









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